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1. Introduction

This report introduces the GMR-Hungary multiregion - multisector policy impact model, which has been developed to facilitate the economic impact assessment of regional development policies. The most recent version of GMR-Hungary changed majorly compared to earlier models in two respects: (1) the economic impact estimation of policies targeting entrepreneurship and interregional knowledge network development became possible with this model version; (2) with this multi-sector model we can estimate the economic impacts of industry sector-related policies. Thanks to these new extensions, GMR-Hungary now bears those important features that open the possibilities to estimate the economic impacts of smart specialization policy (Varga, Sebestyén, Szabó, Szerb 2020, Varga, Szabó, Sebestyén 2020). In section two, we give an overview of the GMR-Hungary model, whereas sections three to six detail the model blocks. Illustrative policy impact simulations are provided in section seven. In the appendix at the end of the report one can find detailed information on the variables, parameters of the model, the list of regions and industries, the detailed explanation of how the interregional input-output matrix was generated with additional information on the interregional SAM used for calibration and the defined regions in the model.

2. GMR-Hungary: an overview

2.1 General features of GMR models

The geographic macro and regional modeling (GMR) framework has been established and continuously improved to support development policy decisions by ex-ante and ex-post scenario analyses. Policy instruments including R&D subsidies, human capital development, entrepreneurship policies or instruments promoting more intensive public-private collaborations in innovation are in the focus of the GMR-approach.

The novel feature of the GMR-approach is that it incorporates geographic effects (e.g., agglomeration, interregional trade, migration) while both macro and regional impacts of policies are simulated. Models frequently applied in development policy analysis are neither geographic nor regional. They either follow the tradition of macro econometric modeling (like the HERMIN model - ESRI 2002), the tradition of macro CGE modeling (like the ECOMOD model – Bayar 2007) or the most recently developed DSGE approach (QUEST III - Ratto, Roeger and Veld 2009). They also bear the common attribute of national level spatial aggregation.

Why does geography get such an important focus in the system? Why the system is called “regional” and “macro” at the same time? Geography plays a critical role in development policy effectiveness for at least four major reasons. First, interventions happen at a certain point in space and the impacts might spill over to proximate locations to a considerable extent. Second, the initial impacts could significantly be amplified or reduced by short run (static) agglomeration effects. Third, cumulative long run processes resulting from labor and capital migration may further amplify or reduce the initial impacts in the region resulting in a change of the spatial structure of the economy (dynamic agglomeration effects). Forth, as a consequence of the above effects different spatial patterns of interventions might result in significantly different growth and convergence/divergence patterns.

“Regions” are spatial reference points in the GMR-approach. They are sub-national spatial units ideally at the level of geographic aggregation, which is appropriate to capture proximate relations in innovation. Besides intraregional interactions the model captures interregional connections such as knowledge flows exceeding the regional border (scientific networking or spatially mediated spillovers), interregional trade connections and migration of production factors.

Important regional dimensions that may crucially determine the growth effects of development policies include the following aspects.

- Regional development programs are built on important *local specificities* (industrial structure, research strengths of the region, size and specialization of human capital etc.).
- Models have to capture the effects of policies on *local sources of economic growth* such as technological progress, investment and employment.
- The models also need to be able to follow those cumulative *agglomeration impacts* such as intensifying localized knowledge spillovers and their feedback mechanisms that may arise as a consequence of policies.
- There are certain additional impacts on the regional economy instrumented by *Keynesian demand side* effects or *Leontief-type intersectoral* linkages.
- Most of the infrastructural programs target better physical *accessibility*. Impacts of these policies on regions that are (directly or indirectly) affected also have to be reflected.
- There are different mechanisms through which policies implemented in certain regions affect other territories such as *interregional knowledge spillovers and trade linkages* and as such these effects also need to be incorporated in model structures.

The "macro" level[1] is also important when the impact of development policies is modeled: fiscal and monetary policy, national regulations or various international effects are all potentially relevant factors in this respect. As a result the model system simulates the effects of policy interventions both at the regional and the macroeconomic levels. With such an approach different scenarios can be compared on the basis of their impacts on (macro and regional) growth and interregional convergence.

The GMR-framework is rooted in different traditions of economics (Varga 2006). While modeling the spatial patterns of knowledge flows and the role of agglomeration in knowledge transfers it incorporates insights and methodologies developed in the geography of innovation field (e.g., Anselin, Varga and Acs 1997, Varga 2000). Interregional trade and migration linkages and dynamic agglomeration effects are modeled with an empirical general equilibrium model in the tradition of the new economic geography (e.g., Krugman 1991, Fujita, Krugman and Venables 1999). Specific macroeconomic theories are followed while modeling macro level impacts.

The first realization of the GMR approach was the EcoRET model built for the Hungarian government for ex-ante and ex-post evaluation of the Cohesion policy (Schalk and Varga 2004). This was followed by the GMR-Hungary model, which is currently used by the Hungarian government for Cohesion policy impact analyses (Varga 2007). GMR-Europe was built in the IAREG FP7 project (Varga, Járosi, Sebestyén 2011, Varga 2017) and further developed in the GRINCOH FP7 and FIRES H2020 projects (Varga, Járosi, Sebestyén, Szerb 2015, Varga, Sebestyén, Szabó, Szerb 2018, Varga, Sebestyén, Szabó, Szerb 2020). One of the recent versions of the models is GMR-Turkey (Varga, Járosi, Sebestyén, Baypinar 2013, Varga and Baypinar 2016).

GMR models reflect the challenges of incorporating regional, geographic and macroeconomic dimensions in development policy impact modeling by structuring the system around the mutual interactions of three sub-models such as the Total Factor Productivity (TFP), the Spatial Computable General Equilibrium (SCGE) and the macroeconomic (MACRO) model blocks.

Some policy interventions can be modeled in the macroeconomic block (such as changes in international trade, in tax regulations or in income subsidies) via policy shocks affecting specific macroeconomic equations. However, many other policy instruments may apply on the regional level, stimulating the regional base of economic growth such as investment support, infrastructure building,

[1] We use the term 'macro' here to refer to the supra-regional level, which is traditionally a national level, but in the GMR Europe model it may be split to national (country) and international (EU) levels.

human capital development, R&D subsidies, promotion of (intra- and interregional) knowledge flows or entrepreneurship development. These interventions are modelled in the regional model blocks and also interact with the macroeconomic part. In the following sub-section we focus on mechanisms of these latter policies.

2.2 Regional impact mechanisms of the main policy variables

2.2.1 R&D support, interregional knowledge networks, human capital and entrepreneurship

Figure 1 shows the way, how the impacts of policies targeting R&D support, interregional knowledge networks, human capital and entrepreneurship are modeled in the GMR model. Economically useful new knowledge is measured by patents in the model. R&D support interregional knowledge networks and agglomeration effects proxied by the size of the regions affect the economy via their impacts on regional patenting. Increasing patenting activity may in turn affect the regions' general technological level which then contribute to higher productivity, captured by total factor productivity (TFP) in the model. Productivity, on the other hand is also affected by the regional level of human capital and the quality of the entrepreneurial environment.

The impacts of the promotion of R&D, networking, human capital and entrepreneurship on economic variables (prices of quantities of inputs and outputs, etc.) are calculated in the SCGE block. Economic impacts of increased productivity are modeled in the SCGE block in the following steps.

2.2.1.1 Short run effects

The impact in the short run results from the interplay between the substitution and output effects. Assuming that the level of production does not change the same amount of output can be produced by less input that is the demand for capital (K) and labor (L) decrease as a result of the interventions. However, increased TFP makes it also possible to decrease prices to keep firms more competitive, which positively affects demand. This latter effect is called the output effect. The interaction of output and substitution effects might result in the increase of the equilibrium utilization of factor inputs (K and L) but also the impact can be just the opposite. What will actually happen is an empirical question. In case output effect exceeds substitution effect wages will increase in the short run, which together with the relative decrease in prices will result in increasing consumption and higher utility levels.

2.2.1.2 Long run effects

Increased utility levels result in in-migration of labor and capital (depending on the change in capital returns, which generally increase with an increasing marginal product of capital) into the region, which will be the source of further cumulative effects working via centripetal and centrifugal forces. Labor migration increases employment concentration, which is a proxy for positive agglomeration effects in the model. According to findings in the literature localized knowledge spillovers intensify with the concentration of economic activity in the region (e.g., Varga 2000). A higher level of employment thus increases TFP (as shown also in Figure 1), which further reinforces in-migration of production factors following the mechanisms described above. However increasing population also affect the average size of per capita living space negatively (given a fixed amount of total flats/living space) which work as a centrifugal force in the model, controlling for congestion in the long run dynamics of the model. The balance between centrifugal and centripetal forces will determine the long term cumulative effect of policies at the regional, interregional and macroeconomic levels.

2.2.2 Private investment support

One of the policies suggested is the support of investment by small and medium sized enterprises. The mechanism of this policy instrument affects the model via the increase in private capital, which has further impacts on several other variables both in the region where the intervention occurs and in other regions connected by trade or migration linkages.

2.3 Macroeconomic impacts

Contrary to earlier versions of GMR models where a separate MACRO block was responsible for the macro economy (e.g., Varga, Sebestyén, Szabó, Szerb 2018) in the most recent version of GMR-Hungary we do not build a complete macroeconomic model into the system. First, dynamics is now modeled in the interrelations of the TFP and the SCGE model blocks. Furthermore, some of the aspects of the economy that were previously integrated in the MACRO block are now incorporated in the SCGE block (i.e., government revenues and expenditures, taxes, exchange rates). What is still modeled in a separate macroeconomic block is the endogenous determination of government deficit.

2.4 Impact mechanisms in the GMR model

Figure 1 shows the interactions of model blocks within the mutually interconnected model system. As mentioned before, the TFP block controls changes in regional productivity levels, which provide the core inputs to the SCGE block. Changes in regional productivity levels then influence the allocation of production factors, production, trade, migration, etc. The SCGE block calculates how regional economic variables respond to these effects, as a result of overall market clearing within and across regions and industries. Economic effects of those policy shocks that enter the model in the SCGE block (i.e., private investment and public infrastructure development subsidies) are also driven by the mutual interactions of the SCGE and TFP model blocks. In addition to changes in several economic variables (like GVA, employment, wages, prices, etc.) induced interregional migration in the next period alters regional employment and as an agglomeration force this affects the level of TFP which then induces further changes in the interconnected model system.

On the other hand, changes in prices, tax revenues, economic growth will have an impact on government spending in the next year calculated by the MACRO block. A change in deficit thus influences current demand of different products through public spending, but on the other hand the deficit must be financed by domestic or foreign savings. As a result, higher deficit will have a considerable investment loss, which influences long-term growth possibilities. We account for labor migration and capital accumulation between two discrete time periods. In the next time period, changes in employment (as a result of the above effects) are channeled back to the regional TFP block accounting for increased positive agglomeration effects in knowledge production. With higher level of regional employment productivity is further improved ceteris paribus, which is then channeled back to the SCGE block and the iteration goes on. As a result of the interactions between each block both supra regional (national) and regional economic impacts are calculated.

As also shown in Figure 1, different policy interventions can be introduced at different levels of the model system. Innovation-related interventions (e.g. R&D support, educational programs, network-development, entrepreneurship programs, etc.) are handled in the TFP block. Region-specific investment support, infrastructural developments are accounted for in the SCGE block, while macro level policies are simulated by the MACRO block (e.g. changes in government spending, tax rates). The direct and indirect effects of all these interventions will flow through the other model blocks and the final economic impacts are determined by the simultaneous interactions between these model components, together with the inner mechanisms of each. As a result, our policy impact simulations are able to track the likely effects of a variety of policy interventions, taking into account complex spatial and inter-industrial interaction mechanisms.

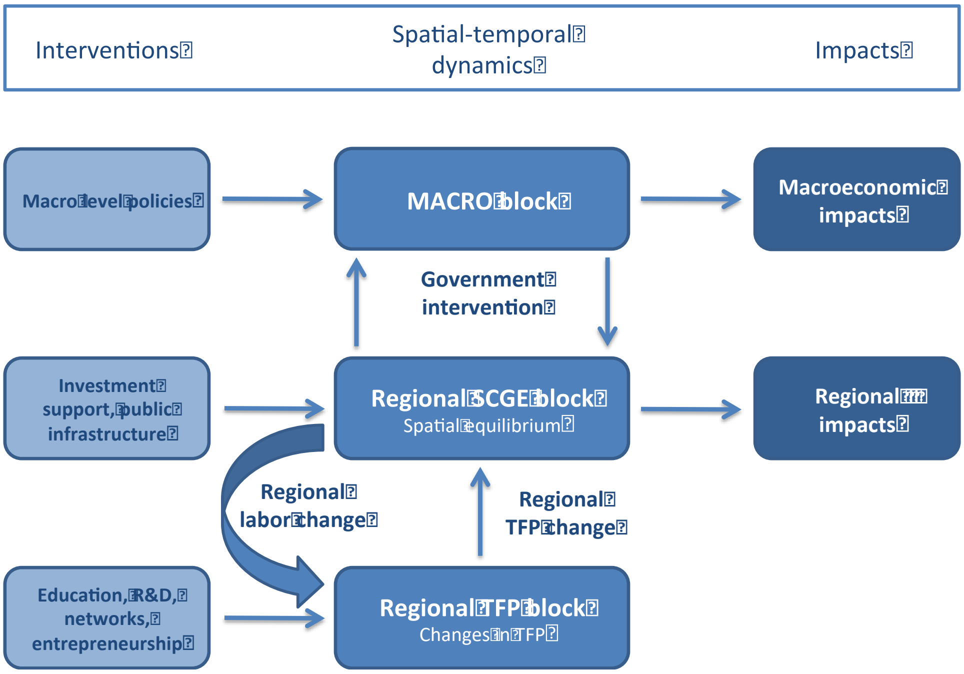

Figure 1: Regional and macroeconomic impacts of the main policy variables in the GMR-Hungary model

3. The Total Factor Productivity (TFP) block

3.1 Estimating TFP for Hungarian counties

Estimating regional TFP values is the first step in the modeling process. The literature provides several ways for the calculation of total factor productivity and there is no consensus among researchers about which one is the most accurate approach (Van Beveren, 2007).

The present model implements an approach based on a Cobb-Douglas type production function:

$$Y_{r,t} = A_{r,t} K_{r,t}^{1-\alpha} L_{r,t}^{\alpha}$$

(1)

where „Y" refers to regional GDP, „K" is the regional capital stock, „L" is regional employment, „α" is the partial output elasticity of labor, „r" is the region index and „t" is the time (year) index. The most crucial variable of the production function is the TFP denoted by „A". Through rearranging equation (1) TFP values could be calculated by evaluating the Solow-residuals for each region and each year as follows:

$$A_{r,t} = \frac{Y_{r,t}}{K_{r,t}^{1-\alpha} L_{r,t}^{\alpha}}$$

(2)

Formula (2) shows that regional GDP, capital, employment and partial output elasticity of labor (or partial output elasticity of capital) are the required variables.

Table 1: Variables and their data sources to calculate TFP at the level of Hungarian counties

Variable	*KSH Stadat data table*
GDP	6.3.1.1. Gross Domestic Product (GDP) (2000–) [regional level, market prices, million Ft] 3.6.1. Consumer Price Index (1985–) [previous year = 100,0%, national level]
Employment	6.2.1.3. Number of employed persons (2000–) [thousands, regional level]
Partial output elasticity of labor	2.2.1.1. Data of total households by deciles, regions and type of settlements (2010-2017) [regional level]
Capital stock	3.1.34. Net stock of fixed assets (1995–) [at market prices, billion Ft, national level] 6.3.3.1. Investments by material-technical category (2008–) [million Ft, regional level] 3.6.23. Price indices of investments by categories (1991–) [national level]

Data for calculating (2) were provided by Stadat tables of the Hungarian Central Statistical Office (KSH). Description of variables are shown in Table 1. To calculate real measures of selected variables we used consumer price index (for GDP) and investment price index (for investment) to deflate the data and convert them to market prices of 2000.

In the estimation of regional capital stocks we followed methodology suggested in Schalk and Varga (2004). First, we calculated average regional investment growth rates between 2001 and 2005. After that regional capital stocks were calculated for the year 2005[2]. The sum of these regional capital stocks for the year 2005 gave us the calculated national capital stock. Then we divided the calculated regional capital stock by the calculated national capital stock to get the regional shares of the national capital stock for the year 2005. Finally, we distributed the net stock of fixed assets in 2005 (provided by the Hungarian Statistical Office) with regard to the calculated regional shares to estimate initial regional capital stocks. After determining the initial regional capital stocks ($K_{2005,r}$), the time series of capital stocks in each region were estimated following the PIM methodology (Hall – Jones, 2009)[3].

Partial output elasticity of labor denoted by „α" in equation (1) is the average ratio of labor income in GDP[4]. With the help of the variables described before regional TFP values were calculated applying formula (2).

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[2] To calculate regional capital stock we used the following formula: $\frac{I_{2005,r}}{g_r+d}$, where the numerator is the investment of the „r"-th region, „g_r" is the investment growth rate of the „r"-th region and „d" denotes capital depreciation.

[3] For the years after 2005 we first depreciated the stock of capital then added investment to the existing capital stock. For the years before 2005 we first subtracted new investment and added depreciation to existing capital stock. Capital depreciation is measured by the ratio between the consumption of fixed capital and the net stock of fixed assets in each year.

[4] According to standard macroeconomic theory, assuming competitive markets, partial output elasticity of labor approximately equals to income-GDP ratio. Similarly, the partial output elasticity of capital approximately equals to capital income-GDP ratio.

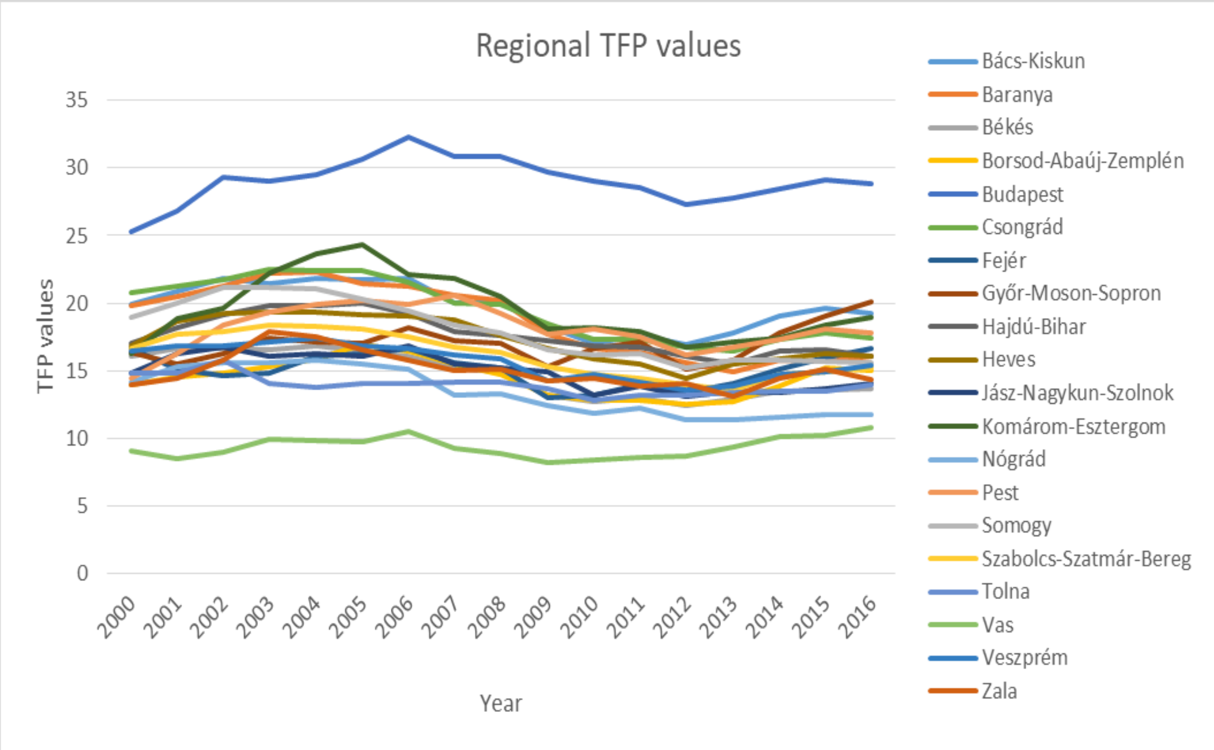

Figure 1: Regional TFP values

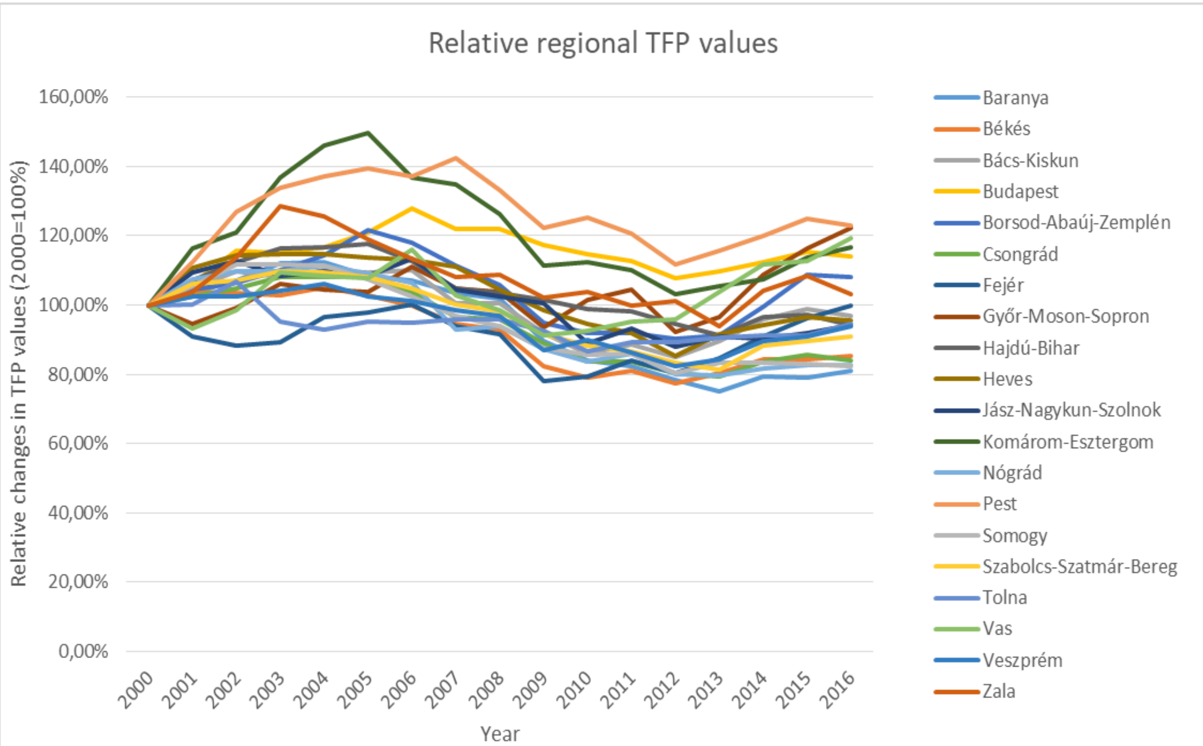

Figure 2: Relative changes in regional TFP values

Table 2: The ranks of regions based on average regional TFP values

Position	Region	Average TFP value
1	Budapest	29.0215004983384
2	Bács-Kiskun	19.7013744271447
3	Komárom-Esztergom	19.5317788985707
4	Csongrád	19.4946067962041
5	Baranya	18.7279238504129
6	Pest	18.0954902255616
7	Somogy	17.9426865168527
8	Hajdú-Bihar	17.6616440794179
9	Heves	17.2987667847144
10	Győr-Moson-Sopron	17.005493763521
11	Szabolcs-Szatmár-Bereg	16.1418178858952
12	Veszprém	15.6325087970476
13	Fejér	15.0735924886187
14	Zala	15.033133973148
15	Jász-Nagykun-Szolnok	14.9169292402197
16	Békés	14.7094978691064
17	Borsod-Abaúj-Zemplén	14.4854384431706
18	Tolna	13.9426011355144
19	Nógrád	13.3913624338644
20	Vas	9.34689751138262

Figures 1 and 2 show the estimated values of regional TFP. Figure 1 represents total TFP values while Figure 2 introduces the changes in relative regional TFP values based on year 2000. The results underpin the prediction one may have related to TFP: the capital city has a leading role in productivity which are followed by regions kept in mind traditionally as developed counties, e.g. Komárom-Esztergom or Pest that have larger TFP values compared to other regions. In conformity with intuitive expectations less developed regions have lower TFP values, for example Borsod-Abaúj-Zemplén or Nógrád. Beyond comparability, each figure emphasizes the impact of financial crisis in 2008-2009. Table 2 indicates the ranks of regions based on average regional TFP values**.** Budapest outstands and regions with „university-cities" (e.g. Baranya, Csongrád and Somogy) and industrial regions (e.g., Komárom-Esztergom, Győr-Moson-Sopron) follow with little differences in their relative TFP values. Surprisingly, region Vas became the last one in the ranking although it cannot be regarded as an under-developed county.

Figure (3) depicts national relative TFP values, which were approximated by the annual regional TFP averages[5]. The tendencies are well perceptible, and the values are consistent with previous findings (Obláth, 2014; Kónya, 2015) calculated directly at the national level.

[5] Weights are created using the ratios between regional GDPs compared to the national GDP reflecting the sizes of the regions.

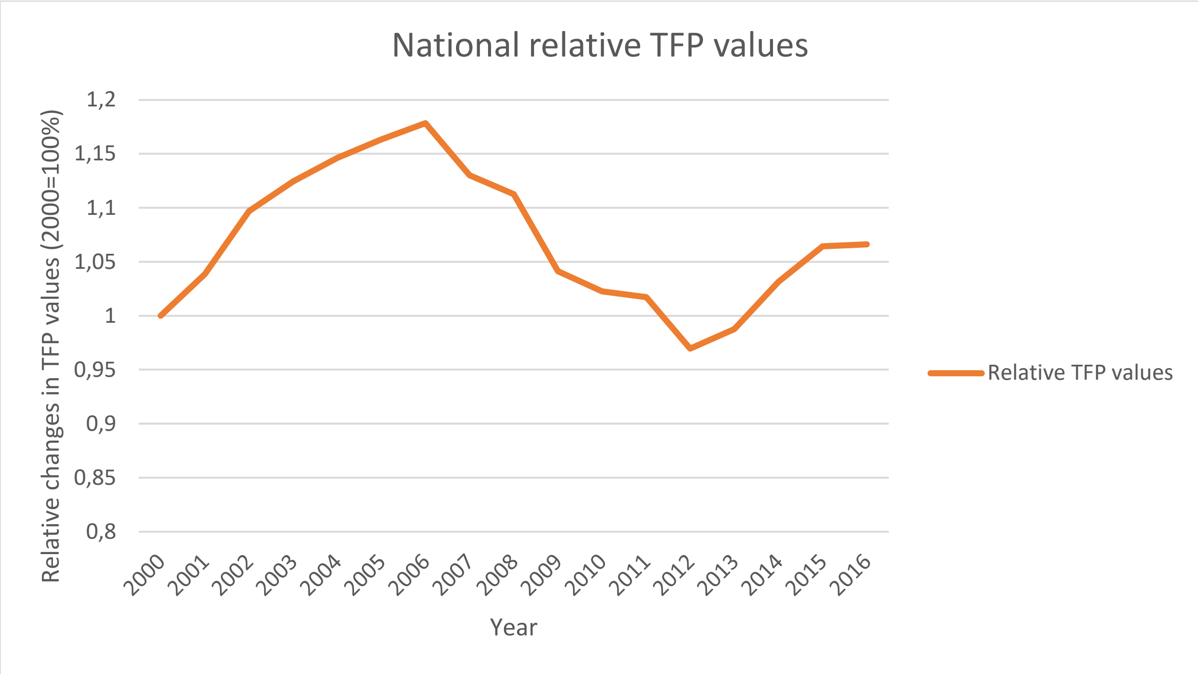

Figure 3: Relative national TFP values

3.2 The setup of the TFP block

Figure 4 shows the structure of the TFP model block. The main focus is to determine regional TFP values, principally as the function of various innovation and entrepreneurial activities. The TFP model block uses a knowledge-production based approach where new knowledge is measured by regional patent activity. Following Romer (1990) new knowledge creation is related to factors of knowledge production indicated by R&D expenditures and already existing knowledge measured by national patent stock. An additional element in our formulation is the quality of research networks appraised by the ENQ-index[6] introduced by Sebestyén and Varga (2013). The ENQ-index measures embeddedness in interregional knowledge networks. We assume that a better quality research network increases the effectiveness of regional research in new knowledge production (Varga, Sebestyén, Szabó, Szerb 2019.) To control for agglomeration externalities the effects of regional employment is estimated in the knowledge production function.

[6] ENQ-index is described as follows: $ENQ^i = \sum_{d=1}^{N-1} W_d LS_d^i KP_d^i$, where „i" is the index of the examined node, „d" is the distance in the network, „N" is the size of the network, „W" is a weight factor, „LS" is the local structure and „KP" is the knowledge potential (see: Sebestyén-Varga (2013)).

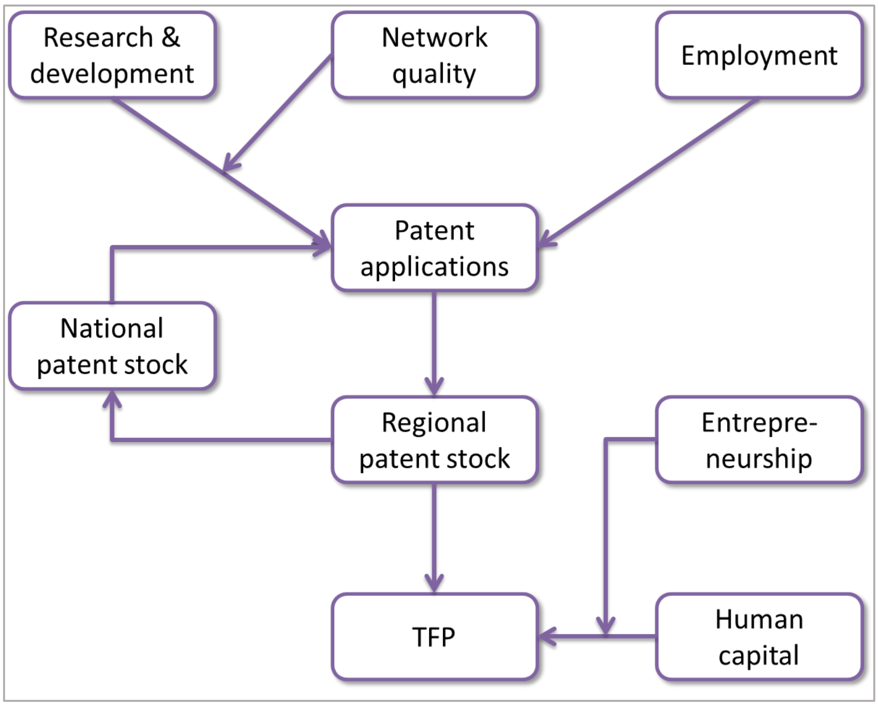

Figure 4: The schematic structure of the TFP model block

Thus, innovation affects TFP directly through regional patent stock. Regional human resources also influence regional productivity. However, human resources may interact with the entrepreneurial environment measured by the REDI-index developed by Szerb et al. (2017). Following the knowledge spillover theory of entrepreneurship (Ács et al. 2008) we assume that a better quality entrepreneurial environment contributes to higher utilization of knowledge and creativity owned by regions' human resources (Varga, Szerb, Sebestyén, Szabó 2020).

3.3 Model equations, variables, estimation and calibration

The TFP model block is based on the following two equations:

$$Regional\ patent\ applications = \\ = f(national\ patent\ stock, R\&D\ expenditures, ENQ, labour\ force) \quad (3)$$

$$Regional\ TFP = f(regional\ human\ resources, REDI, regional\ patent\ stock) \quad (4)$$

Equations 3 and 4 are connected through the knowledge production process explicitly. Equation 3 estimates the amount of new patents per year for each region. New knowledge affects TFP since new technologies broaden production possibilities. This effect is delivered through patent stock values into TFP. The role of agglomeration externalities and human resources in productivity also appear in equations 3 and 4.

Relationships represented by equations 5 and 6 are econometrically estimated as follows:

$$\log(Patent)_{r,t} = \beta_0 + \beta_1\log(RD)_{r,t-1} + \beta_2\log(RD)_{r,t-1} * \log(ENQ)_{r,t-1} + \beta_3\log(PatStock)_{r,t-1} + \beta_4\log(EMP)_{r,t-1} + \beta_5 SoDum_{r,t} + \beta_6 SzaDum_{r,t} + \varepsilon_{r,t} \quad (5)$$

$$\log(TFP)_{r,t} = \beta_0 + \beta_1\log(HumCap)_{r,t-1} + \beta_2\log(HumCap)_{r,t-1} * REDI_{r,t-1} + \beta_3\log(RegPatStock)_{r,t-1} + \beta_4 BpPeDum_{r,t} + \beta_5 RegPatStock_{r,t-1} * BpPeDum_{r,t-1} + \beta_6 KeDum_{r,t} + \beta_7 VaDum_{r,t} + \vartheta_{r,t}$$

(6)

Variables referring to the main effects and interaction terms are also included in equation 5 and 6. In equation 5 the multiplicative term reflects the extent to which quality of the research network influences the effectiveness of R&D expenditures. Similarly, in equation 6 the interaction term estimates the extent to which the quality of the entrepreneurial environment affects the utilization of human resources[7]. Regarding the further terms in the equations, „ε" is the patent equation error term, „ϑ" is the TFP equation error term, „r" is the index of the region, while „t" is the time index. The TFP model block is built by these two equations thus its structure is also determined by them. The variables of equation 5 and 6 and the data sources are reported in Table 3.

After estimating the two equations of the TFP block (the TFP equation and the patent equation), we have a system of equations which is able to simulate the effects of different interventions affecting research and development, human capital, networking or the entrepreneurial climate on regional TFP. One drawback of this system is that the estimated coefficients, which drive these impacts are common across all regions in the model, reflecting average tendencies in the sample of regions. However, one may argue that due to differences in the development level of Hungarian regions, mechanisms through which different interventions affect regional productivities differ largely across regions.

We control for these differences in two ways:

- First, in both equations the interaction terms render the respective marginal effects of R&D, human capital, network quality and entrepreneurship development level regions-specific.
- Second, we augment this heterogeneity with a specific calibration process through which region-specific parameters are calculated through an optimization process to improve model fit. This second method is discussed briefly in what follows.

To obtain long-run equilibrium values of the variables we estimated the baseline values of the above-written variables except regional and national patent stocks. Given the observed data, we fit linear trends on these data points for all variables, except regional and national patent stocks (the former is directly given by equation 5 and the latter is calculated by summing up regional patent stocks in each period). After trend fitting, we extrapolate the trend for out-of-sample years. These trends constitute the baseline of the TFP block. After having the extrapolated trend values for all variables in the TFP block (except regional and national patent stocks), we run the regressions in equations 5 and 6 on these data points as well. Coefficients estimated on the historical data and coefficients estimated on the trend data stay fairly close to each other. Table 3 lists variables and their data sources of the TFP equations and Table 4 presents regression results.

[7] The econometrics literature usually suggests to use not only the interactive term but also the main effects represented by the variables included in the interaction. However, there are occasions when one may depart from this rule. One special case is when the variables are independent of each other when one of the interactives equals zero (Nelder, 1998). This condition is met in both equations. If the variable R&D expenditures takes the value of zero, its relation with ENQ disappears. Similarly, if human resources takes the value of zero, the effect of the REDI will also disappear. Therefore, following Nelder (1998) in equations 5 and 6 interactions are allowed to apply with using only one of the main effects.

Table 3: Variables of the TFP model block and their data sources

Variable	*Description of variable*
Patent	Number of new patents per region based on OECD PatStat database
RD	Regional R&D expenditures, calculation based at 2000's market prices. Source KSH (Hungarian Central Statistical Office)
ENQ	Regional ENQ-indexes. ENQ-indexes are calculated to NUTS2 regions and assigned to appropriate NUTS3 regions.
PatStock	National patent stock. Calculated by using depreciation rate of 13%[8] and fitted to the sum of regional patent stocks.
EMP	Regional employment, source: KSH
HumCap	Regional human resources. Measured by ratio of tertiary educated people in each region. Source: KSH.
REDI	REDI index calculated to each region, source: Szerb et al. (2017)
RegPatStock	Regional patent stock. Calculated by using PIM method employed 13% depreciation rate, fitted to former calculations.
SoDum	Dummy of region Somogy. It takes the value of one if the observation refers to county Somogy, 0 otherwise.
SzaDum	Dummy of region Szabolcs-Szatmár-Bereg. It takes the value of one if the observation refers to county Szabolcs-Szatmár-Bereg, 0 otherwise.
BpPeDum	Common dummy of regions Budapest and Pest. It takes the value of one if the observation refers to either capital Budapest or county Pest, 0 otherwise.
KeDum	Dummy of region Komárom-Esztergom. It takes the value of one if the observation refers to county Komárom-Esztergom, 0 otherwise.
VaDum	Dummy of region Vas. It takes the value of one if the observation refers to county Vas, 0 otherwise.

[8] The13% depreciation rate comes from international experiences (Varga, Pontikakis, Chorafakis 2014).

Table 4: Econometric estimation results of the TFP model block

Patent equation		TFP equation	
Variable	Coefficient	Variable	Coefficient
Constant	-21.836***	Constant	1.618***
	(1.236)		(0.134)
RD	0.243***	HumCap	0.076***
	(0.046)		(0.012)
RD*ENQ	0.27*10-10***	HumCap*REDI	0.001***
	(0.000)		(0.000)
PatStock	0.421**	RegPatStock	0.049***
	(0.170)		(0.005)
Emp	1.489***	BPPEDUM	-1.243***
	(0.057)		(0.069)
SoDum	-0.339***	RegPatStock*BPPEDUM	0.214***
	(0.080)		(0.014)
Szadum	-0.847***	KEDUM	0.250***
	(0.142)		(0.019)
		VADUM	-0.542***
			(0.020)
Adj. R^2	0.894	Adj. R^2	0.842
Observations	220	Observations	280

$$
\begin{aligned}
min: &\sum_{r=1}^{20}\sum_{t=2017}^{2030}\left|\frac{Patent_{rt}-\widetilde{Patent}_{rt}}{\widetilde{Patent}_{rt}}\right| + \\
&+\sum_{r=1}^{20}\sum_{t=2017}^{2030}\left|\frac{TFP_{rt}-\widetilde{TFP}_{rt}}{\widetilde{TFP}_{rt}}\right| + \\
&+\sum_{r=1}^{20}\sum_{t=2017}^{2030}\left|\frac{RegPatStock_{rt}-Reg\widetilde{PatStock}_{rt}}{Reg\widetilde{PatStock}_{rt}}\right| + \\
&+\sum_{r=1}^{20}\left|\frac{Patent\_Constant_{r}-Patent\_\widetilde{Constant}_{r}}{Patent\_\widetilde{Constant}_{r}}\right| + \\
&+\sum_{r=1}^{20}\left|\frac{TFP\_Constant_{r}-TFP\_\widetilde{Constant}_{r}}{TFP\_\widetilde{Constant}_{r}}\right|
\end{aligned}
\qquad (7)
$$

The coefficients estimated econometrically on the trend data constitute the basis of parameter calibrations in the next step. The aim of the calibration is to find region-specific values for selected parameters, which improve the overall fit of the model while they meet certain conditions. After a careful selection procedure among several model versions three coefficients of the TFP block, namely the constant term and the coefficient of patent stock in the patent equation and the constant term in the TFP equation are calibrated. Thus the model was recalibrated until we reached the possible minimum of the constructed error function familiarized by equation (7). As a result of this calibration process, we end up with region-specific parameter values for the listed three parameters of the TFP block which improve the fit of the TFP block equations and retain the average tendencies represented by the trend-based estimation. Tables 5 and 6 summarize the coefficients of the TFP block.

Table 5: Coefficients of the Patent equation and their calculation

Coefficient	*Level of specification*
β_0	region-specific (calibrated)
β_1	universal (econometrically estimated)
β_2	universal (econometrically estimated)
β_3	universal (econometrically estimated)
β_4	universal (econometrically estimated)
β_5	universal (econometrically estimated)
β_6	universal (econometrically estimated)

Table 6: Coefficients of the TFP equation and their calculation

Coefficient	*Level of specification*
β_0	region-specific (calibrated)
β_1	universal (econometrically estimated)
β_2	universal (econometrically estimated)
β_3	region-specific (calibrated)
β_4	universal (econometrically estimated)
β_5	universal (econometrically estimated)
β_6	universal (econometrically estimated)
β_7	universal (econometrically estimated)

3.4 Sensitivity analysis

We assess the robustness of the TFP model equations with a sensitivity analysis. A question arises here is whether changes in initial circumstances represented by changing coefficients of key policy variables (R&D expenditures, ENQ, human resources, REDI) affect model performance. Our sensitivity assessment is the end-result of several simulations. We changed the coefficient of each policy variable in both directions by 5%[9]. These changes were introduced in all of the possible combinations of the parameters. The two TFP model equations have been then re-estimated with the changed parameters repeatedly. Firstly MAPE[10] values of the original TFP and Patent equations and secondly TFP and Patent values of the original equations are compared to their respective simulated values.

The above-written sensitivity analysis methodology was carried out step-by-step as follows. As the first step we increased the coefficient of R&D expenditures artificially by 5% and re-estimated the model. MAPE values of the simulated equations on the one hand and TFP and Patent values of the simulated equations on the other were compared to their respective original ones. According to our approach if there are no changes (or the changes are moderate) in MAPE values of the simulated equations compared to the original ones, model set-up is sufficiently robust for impact assessment. Similarly, if there are no changes (or the changes are moderate) in TFP and Patent values of the simulated equations compared to the original ones, we regard the model set-up as sufficiently robust for impact assessment. Thus, sensitivity analyses were performed in two different ways simultaneously.

After evaluating the differences in MAPEs and estimated patent and TFP values, the coefficient of R&D expenditures was set back to its original value. We continued the sensitivity analysis through the artificial 5% increase in the coefficient of ENQ executing the same methodology. As the third step, the

[9] The choice of a 5% change in parameter values is a frequently applied solution in the practice of sensitivity assessment (Gujarati, 2003).
[10] Since MAPE is normalized and express relative relations we apply this measure for sensitivity analysis.

coefficients of R&D and ENQ were increased jointly. Following this way we re-estimated the TFP block with each combination of the increased regression coefficients (until each policy variable coefficient was jointly increased by 5%). The sensitivity analysis then was continued in an analogous way with decreased policy variable coefficients.

Figure 5 documents the results of the sensitivity assessment based on MAPE values while figure 6 represents the TFP-Patent based results. Evaluating robustness 30 different models with modified parametrization were examined (15 through increases in initial regression coefficients and 15 through decreases in initial regression coefficients). In the case of MAPE measures 20 - 20 values were compared to the respective original ones based on TFP – Patent values. Thus overall 1200[11] MAPE values were available for sensitivity analysis. As it is shown in figure 5 differences in MAPE values are quite small. The minimum of differences is -3.56% meanwhile the maximum equals to 9.38%. However, the majority of divergence is lower than or equal to 1%: in 1130 cases from all of the 1200 the above mentioned result holds. The divergence exceeds 1% just in 70 cases. Figure 5 also evidences that in the case of increased coefficients differences are rather negative, meanwhile in the case of decreased parameters they are rather positive.

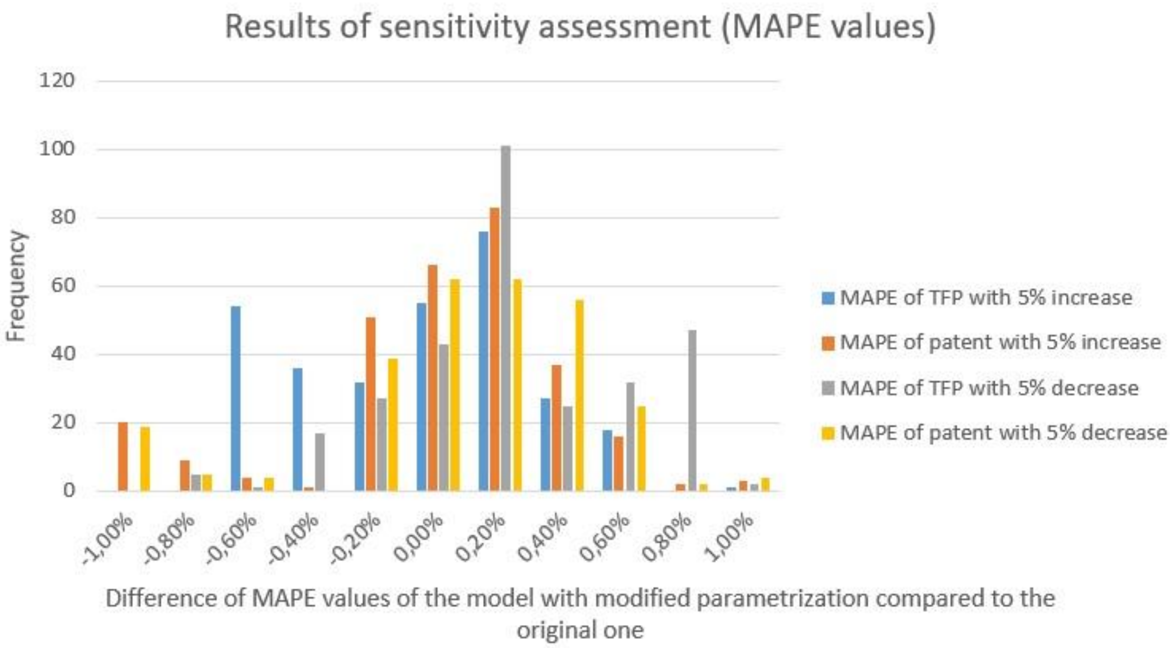

Figure 5: Sensitivity assessment results of MAPE values

Regarding the TFP and Patent paths 280 values were available for every single modified parametrization.[12] 8400 re-estimated TFP and 8400 re-estimated patent values were compared to the respective original model values.[13] Histograms of Figure 6 indicate that the TFP model is strongly robust to parameter changes. In 16113 cases the difference was between -1% and 1% which means that the

[11] 20 MAPE values based on the TFP equation and 20 MAPE values based on the Patent equation in each modified case. Regarding the 15 different parametrizations, overall 40*15=600 MAPE values were available with the increases in the parameters, and 600 with the decreases in the parameters.

[12] As Equation 7 shows the time-window of minimization is 2017-2030 which means 14 time periods. Thus 20 TFP and 20 Patent values for each region gave us 280 TFP and 280 Patent values for every single modified parametrization.

[13] 280 values for each model variant served us 280*15*2=8400 TFP and 8400 Patent values altogether.

difference is lower than or equal to 1% in the 95.91% of the cases. This includes those 8157 cases where the divergence was lower than 0.001% (i.e., in the 48.55% of all cases).

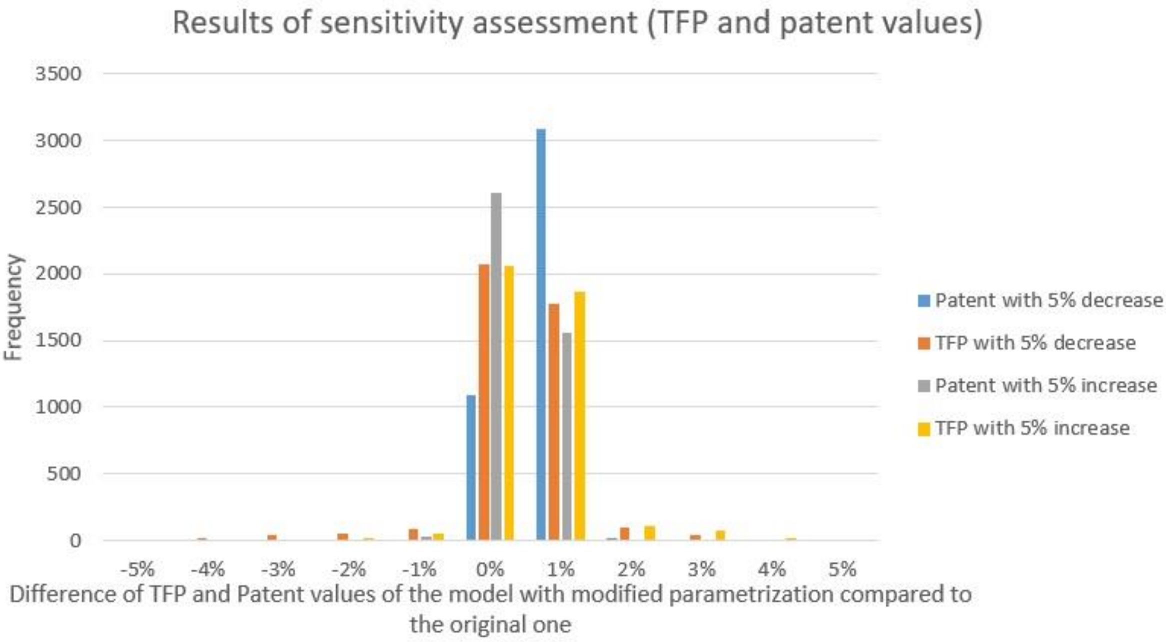

Figure 6: Sensitivity assessment results of TFP and Patent values

On the whole, it makes no difference whether goodness-of-fit measures or response variables are regarded, the model set-up is strongly robust to the changes in initial exogenous coefficients.

4. The SCGE block

In this section we describe the structure of the spatial computable general equilibrium (SCGE) model block of the multiregional - multisector GMR-Hungary model. Since geography plays an important role in the determination of economic impacts of different policy interventions, many spatial aspects of the economy are integrated into the modelling system. First, regions are interlinked through interregional trade of goods and services, labor migration and capital flows. Second, interregional trade is influenced by industry-specific transportation cost. Third, agglomeration externalities (positive and negative) are also taken into account. Apart from these features the model is formalized as a standard CGE model.

Production is modelled according to a multilevel Johansen type production function (Johansen, 1960) as follows. Industries use labor and capital to produce a composite production factor according to a CES technology. This composite factor is then used in fixed proportion with a composite intermediate good to produce the local industrial output (excluding imported inputs) in which the composite intermediate good is a Leontief composite of different industrial inputs. In other words, any industry uses intermediates of all other industries according to fixed coefficients. However these intermediates can be imported from any of the regions through interregional trade. This local output (excluding imported inputs) is then combined with foreign imported inputs to produce gross regional output according to CES (Armington) specification which means the domestic production (the composite of primary factors and intermediate inputs) can be substituted by imported products in production.

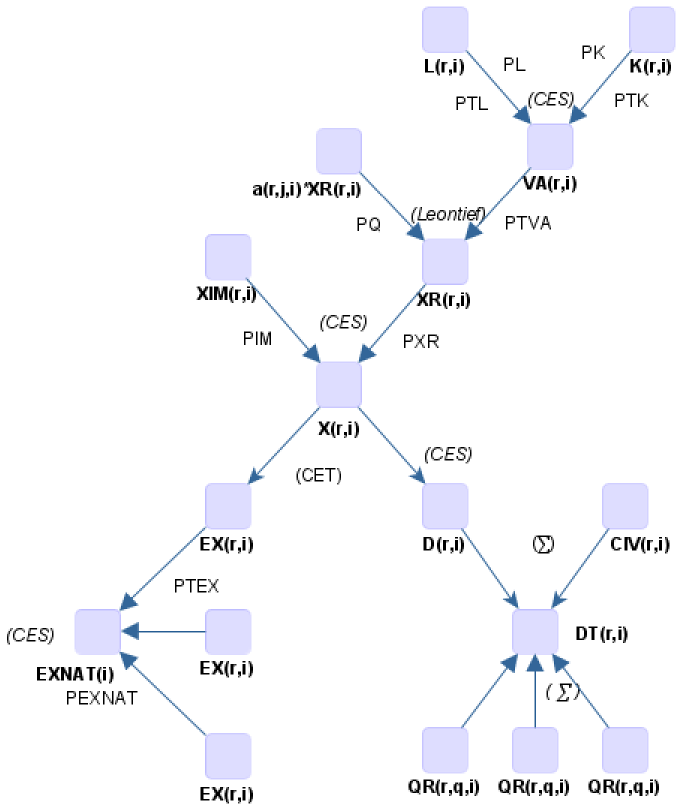

Figure 7: The production structure of the model

Total regional output ($XT_{r,i}$) is a composite of another input, the changes of inventories and valuables. This is due to the complications negative changes of inventories could cause on the demand side. Since in some cases it is possible the negative changes in inventories (the use of goods produced in previous time periods) would create negative interregional demand which is not consistent with the logic of any equilibrium-based model. Thus we decided to account for the changes in inventories on the supply side, thus a negative change in inventories would increase total supply (detailed description can be found in Section 4.1.4.). However, mathematically there is still a risk that a positive change in inventories would decrease regional supply too much since the amount of changes in inventories is usually not significant. Finally, at top level gross output is split into domestic and export supply according to CET technology. The domestic supply can satisfy domestic demand in any regions through interregional trade. For the visual illustration of production technology see Figure 7.

In this model version the market for goods and services is defined as a regional and not as an interregional market which means that the destination region of domestic supply is determined by the demand side. At this point the interregional trade comes into play. The regional output can be used in any regions for different goals (intermediate use, consumption, investment, government consumption) but a given amount of the transported goods and services "melts" in the trade process as a result of iceberg type transportation cost (Samuelson, 1954). In other words, interregional trade is determined on the demand side and regional domestic supply is assumed to satisfy it in equilibrium.

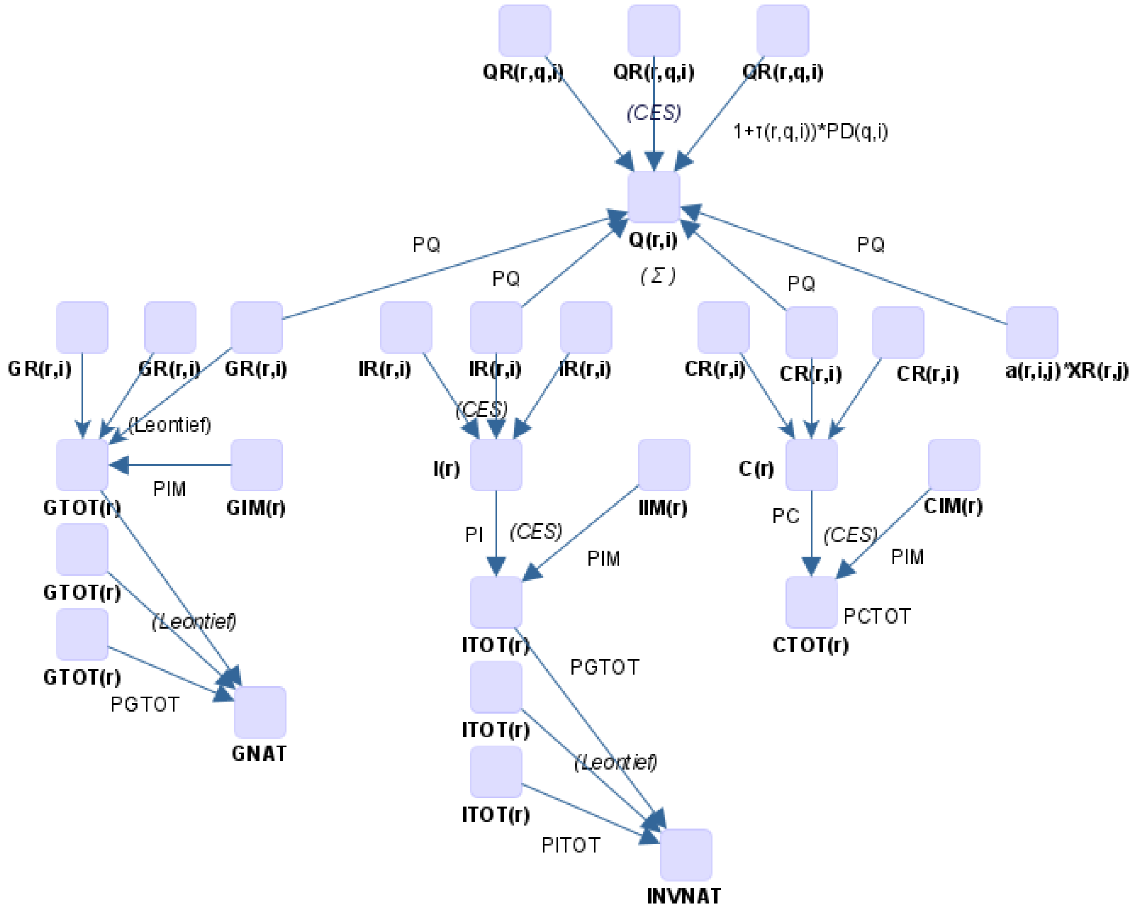

Figure 8: The demand side of the model

Although transportation could be modelled in the form of separate regional transportation industries that can take care of transportation of different products between each region pair. This would be theoretically appealing but on the other hand computational cost of such a complex model would increase dramatically. As a result, we decided to employ a more practical and widely employed approach of interregional trade, the iceberg-type transportation cost.

On the demand side, households spend their income of consumption and they can make savings. Their income consists capital income and wages. Central (and regional) government earns tax revenues (after commodity and production taxes) which can be spent on goods and services. The government can also make savings which is usually negative. Investment is then financed by the total saving made by all actors in the economy (including foreign saving determined by the trade balance equation of the model). Finally, the last element of regional domestic demand for goods and services is intermediate demand by industries. The total demand is then can be satisfied by the production of industries located in any regions of the country. Apart from domestic supply, all final users are allowed to substitute domestic goods and services with foreign imported products. For a detailed visualization of the demand side of the model see Figure 8.

Finally, Figure 9 explains the income flows of the model, starting from factor incomes and tax revenues and ending with the spending on final goods and savings.

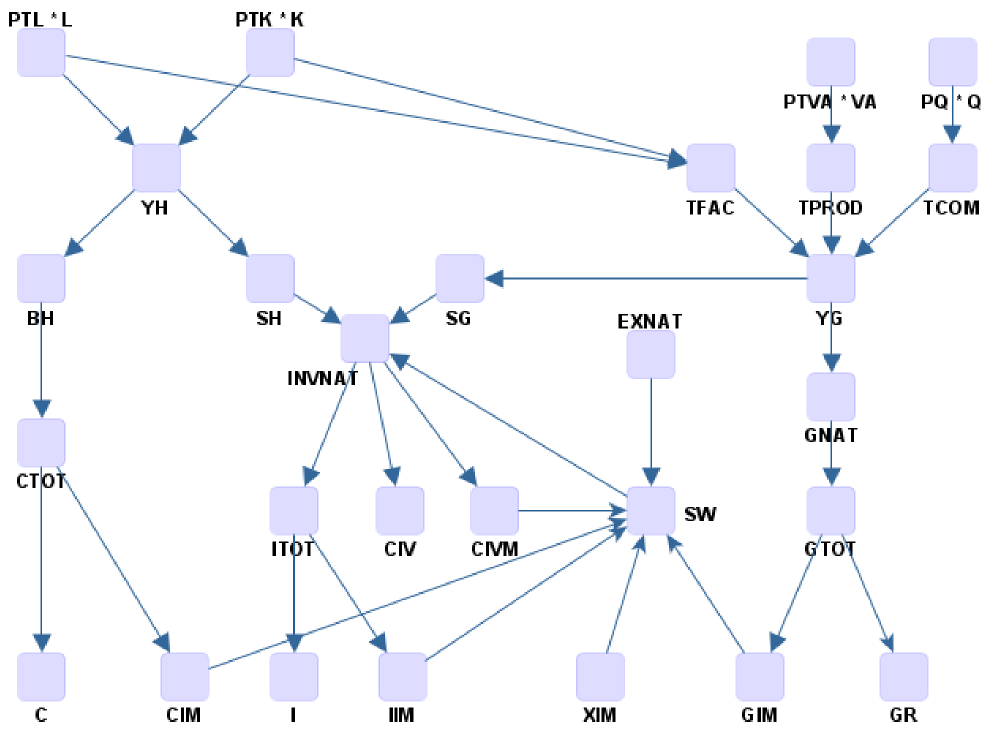

Figure 9: Income flows in the model

In the model description below, we use the following notation. Upper case letters are used to denote endogenous variables and lower case letters for parameters. The indices r and q are used to refer to regions whereas indices i and j are used to refer to industries.

4.1. The production of industries

In this section we describe the production structure of industries which is represented by a multilevel nested production function. Substitution and transformation elasticities are set exogenously and all the other parameters are calibrated based on the estimated interregional input-output table.

4.1.1. Composite factor or value added

The first step of production combines labor and capital (both specific to the regions) into a composite production factor or value added:

$$VA_{r,j} = aCD_{r,j} \cdot L_{r,j}^{\alpha_{r,j}} \cdot K_{r,j}^{\beta_{r,j}} \tag{8}$$

where $VA_{r,j}$ is the composite factor[14] used in industry j in region r, $L_{r,j}$ is the labor demand and $K_{r,j}$ is the capital demand, $\alpha_{r,j}$ and $\beta_{r,j}$ are the share parameters of factors and $aCD_{r,j}$ is the industry level TFP. From this production function, cost minimization yields the following demand functions for labor and capital:

[14] Note that gross value added is not perfectly consistent with this composite factor for two reasons: 1) net production taxes are accounted for in a later step which is a part of value added, 2) households' own activities are not accounted for as sector thus it is not contained by the composite factor.

$$L_{r,j} = \left(\frac{VA_{r,j}}{aCD_{r,j}}\right) \cdot \left(\frac{\alpha_{r,j} \cdot PTK_{r,j}}{\beta_{r,j} \cdot PTL_{r,j}}\right)$$

(9)

$$K_{r,j} = \left(\frac{VA_{r,j}}{aCD_{r,j}}\right) \cdot \left(\frac{\beta_{r,j} \cdot PTL_{r,j}}{\alpha_{r,j} \cdot PTK_{r,j}}\right)$$

(10)

where $PVA_{r,j}$ (the price index of the composite factor) is the shadow price of the composite factor showing us the improvement of the objective function of the optimization problem if the constraint is extended by 1 unit. $PTL_{r,j}$ and $PTK_{r,j}$ are the unit cost of the labor and capital (including factor taxes) respectively. Since in our model we only introduce equations as constraints (no inequalities), the derivation of the demand functions (and later supply functions) is based on the traditional Lagrange method. Finally, the total cost of primary factors is given by Equation 11:

$$PVA_{r,j} \cdot VA_{r,j} = PTL_{r,j} \cdot L_{r,j} + PTK_{r,j} \cdot K_{r,j}$$

(11)

The relationship between unit costs, taxes included and net prices (taxes excluded) are described below:

$$PTL_{r,j} = (1 + tlab_{r,j}) \cdot PL_{r,j}$$

(12)

$$PTK_{r,j} = (1 + tcap_{r,j}) \cdot PK_{r,j}$$

(13)

where $tlab_{r,j}$ and $tcap_{r,j}$ are the tax rates on labor and capital incomes respectively. Note that tax rates may be region and sector specific: sector specific rates are realistic, region specific rates may ease the calibration. $PL_{r,j}$ and $PK_{r,j}$ are the unit labor and capital incomes, taxes excluded. Where both factors price is industry and region specific since we assume that both interregional and interindustry mobility of primary factors is imperfect.

Finally, we assume that production taxes are added to the price of the composite factor since the level of production is best represent by this factor. Upper levels in the production function are influenced by other factors (such as foreign and interregional imported goods). The relationship between pre- and post-tax prices is simply defined as:

$$PTVA_{r,j} = (1 + tprod_{r,j}) \cdot PVA_{r,j}$$

(14)

where $tprod_{r,j}$ is the region- and sector-specific production tax rate.

4.1.2. Local industrial production

The composite factor is merged with intermediates using a Leontief technology:

$$XR_{r,j} = min\left(\frac{1}{a_{r,1,j}} \cdot XIR_{r,1,j}, \frac{1}{a_{r,2,j}} \cdot XIR_{r,2,j}, \dots, \frac{1}{a_{r,3,j}} \cdot XIR_{r,3,j}, \frac{1}{a_{r,j}^{VA}} \cdot VA_{r,j}\right)$$

(15)

where $XR_{r,j}$ is the local production of activity *i* of in regional *r* (which will be later merged with imported inputs) and $XIR_{r,\mathrm{i},j}$ is the use of intermediate products I by industry j, $a_{r,i,j}$ and $a_{r,j}^{VA}$ are the coefficients defining the requirement of composite factors and intermediates in order to produce one unit of output. The demand function for composites factor is given by following equation:

$$VA_{r,j} = a_{r,j}^{VA} \cdot XR_{r,j}$$

(16)

Finally, the total cost of using primary and intermediate inputs is defined by the following equation:

$$PXR_{r,j} \cdot XR_{r,j} = PTVA_{r,j} \cdot VA_{r,j} + \sum_{i} (1 + tcomXIR_{r,j}) \cdot PQ_{r,i} \cdot a_{r,i,j} \cdot XR_{r,j}$$

(17)

where $PTVA_{r,j}$ is the price index of the composite factor (including taxes on production), $PQ_{r,i}$ is the price index of domestic supply of product *i*, which is increased by the industry specific commodity tax rate ($tcomXIR_{r,j}$).

4.1.3. Regional industry output

The local production activity $XR_{r,j}$ is merged with imported inputs through a CES aggregator, resulting in regional industry output $X_{r,j}$:

$$X_{r,j} = d_{r,j}^{X1} \cdot \left[b_{r,j}^{XR} \cdot \left(XR_{r,j}\right)^{\rho_{r,j}^{X1}} + b_{r,j}^{XIM} \cdot \left(XIM_{r,j}\right)^{\rho_{r,j}^{X1}}\right]^{\frac{1}{\rho_{r,j}^{X1}}}$$

(18)

where $X_{r,j}$ is the composite regional output, $XR_{r,j}$ is the local production activity and $XIM_{r,j}$ is the import (import is not broken down to commodities, j here refers to the industry producing). $b_{r,j}^{XR}$ and $b_{r,j}^{XIM}$ are the share parameters, $d_{r,j}^{X1}$ is the shift parameter, while $\rho_{r,j}^{X1}$ is the substitution parameter. From this production function, cost minimization yields the following demand functions:

$$XR_{r,j} = \left(\frac{PX_{r,j}}{PXR_{r,j}}\right)^{\sigma_{r,j}^{X1}} \cdot \left(b_{r,j}^{XR}\right)^{\sigma_{r,j}^{X1}} \cdot \left(d_{r,j}^{X1}\right)^{\sigma_{r,j}^{X1}-1} \cdot X_{r,j}$$

(19)

$$XIM_{r,j} = \left(\frac{PX_{r,j}}{PIM_r}\right)^{\sigma_{r,j}^{X1}} \cdot \left(b_{r,j}^{XIM}\right)^{\sigma_{r,j}^{X1}} \cdot \left(d_{r,j}^{X1}\right)^{\sigma_{r,j}^{X1}-1} \cdot X_{r,j}$$

(20)

where $PX_{r,j}$ (the net price index of the regional output) is again the shadow price of the optimization problem, $PXR_{r,j}$ and PIM_r are the price indices of the local industrial activity and import respectively, $\sigma_{r,j}^{X1}$ is the elasticity of substitution which is linked to the substitution parameter: $\rho_{r,j}^{X1} = (\sigma_{r,j}^{X1} - 1)/\sigma_{r,j}^{X1}$. The total cost of using domestic and imported inputs is defined by the following equation:

$$PX_{r,j} \cdot X_{r,j} = PXR_{r,j} \cdot XR_{r,j} + PIM_r \cdot XIM_{r,j}$$

(21)

Note, that we allow for different import prices to prevail in different regions. This may reflect transportation costs, but the prices may be set equal across regions if necessary. In the current model setup these import prices are given by the multiplication of exogenous world prices and endogenous exchange rate (see Equation 81).

4.1.4. Transformation of output into supply

Once total output is produced it is first split between export and domestic supplies according to a CET technology:

$$X_{r,j} = d_{r,j}^{X2} \cdot \left[b_{r,j}^{EX} \cdot \left(EX_{r,j}\right)^{\rho_{r,j}^{X2}} + b_{r,j}^{D} \cdot \left(D_{r,j}\right)^{\rho_{r,j}^{X2}}\right]^{\frac{1}{\rho_{r,j}^{X2}}}$$

(22)

where $EX_{r,j}$ is the exported quantity, $D_{r,j}$ is the domestic quantity. $d_{r,j}^{X2}$ is the shift parameter whereas $b_{r,j}^{EX}$ and $b_{r,j}^{D}$ are the respective share parameters. $\rho_{r,j}^{X2}$ is the transformation parameter. From this CET function, the supply for export and domestic use can be derived as follows:

$$EX_{r,j} = \left(\frac{PEX_{r,j}}{PX_{r,j}}\right)^{\sigma_{r,j}^{X2}} \cdot \left(b_{r,j}^{EX}\right)^{-\sigma_{r,j}^{X2}} \cdot \left(d_{r,j}^{X2}\right)^{-\sigma_{r,j}^{X2}-1} \cdot X_{r,j}$$

(23)

where $PEX_{r,j}$ is the export price index, $\sigma_{r,j}^{X2}$ is the elasticity of transformation, determined by the transformation parameter: $\sigma_{r,j}^{X2} = 1/(\rho_{r,j}^{X2} - 1)$. The domestic supply of industry output is determined by the next equation:

$$D_{r,j} = \left(\frac{PD_{r,j}}{PX_{r,j}}\right)^{\sigma_{r,j}^{X2}} \cdot \left(b_{r,j}^{D}\right)^{-\sigma_{r,j}^{X2}} \cdot \left(d_{r,j}^{X2}\right)^{-\sigma_{r,j}^{X2}-1} \cdot X_{r,j}$$

(24)

where $PD_{r,j}$ is the price index of the regional supply for domestic use. Finally the marketed output value is given by the sum of the values of domestic sales and exports:

$$PX_{r,j} \cdot X_{r,j} = PEX_{r,j} \cdot EX_{r,j} + PD_{r,j} \cdot D_{r,j}$$

(25)

As we mentioned it before changes in inventories can vary between negative and positive values and thus as a component of final demand it can change the sign of total interregional final demand and thus interregional trade volumes which can create problems for the model since standard CES and CET functions do not work properly with negative quantities. As a solution we decided to account for the domestic changes in inventories (with a negative sign) as additional supply to the domestic economy and positive changes in inventories (inventory accumulation) as a decrease in current output since these quantities are not sold in the current time period.

$$DT_{r,j} = D_{r,j} + CIV_{r,j}$$

(26)

Where $DT_{r,j}$ can be considered as total composite domestic supply in region *r* in time period t (including current production and sales from previous inventories, but not containing current increase of inventories), $CIV_{r,j}$ stands for the exogenously fixed domestic industrial changes in inventories and valuables. However, this solution can cause distortion since changes in inventories are defined as the change in the value of purchased and produced products stored in the inventories in a given time period, which means that inventories do not concern explicitly the inventories of the output which would be the ideal. By treating both the changes in domestic and imported (see later) inventories exogenously in the current model version we hope that this potential distortion is not significant.

4.2. Demand

This section provides a brief description of the demand side of the model including the final users (households, government, investment, rest of world). Although there are many similarities between the structure of each final user we made special distinction in case of the government assuming a fixed expenditure structure since we believe that in case of public consumption substitution between different goods and services is not a realistic assumption.

4.2.1. Consumption demand

On the first level, households consume a composite consumption good, the price level of which is defined by the following equation:

$$PCTOT_r \cdot CTOT_r = BH_r$$

(27)

where $CTOT_r$ is the composite (real) consumption of households and $PCTOT_r$ is its price index. Composite consumption is composed of domestic and imported goods under the following CES technology:

$$CTOT_r = d_r^{CTOT} \cdot \left[b_r^C \cdot (C_r)^{\rho_r^{CTOT}} + b_r^{CIM} \cdot (CIM_r)^{\rho_r^{CTOT}} \right]^{\frac{1}{\rho_r^{CTOT}}}$$

(28)

where C_r is consumption from domestic sources, CIM_r is imported consumption, d_r^{CTOT} is the shift parameter, b_r^C and b_r^{CIM} are the share parameters and ρ_r^{CTOT} is the substitution parameter. This aggregator gives rise to the following demand functions:

$$C_r = \left(\frac{PCTOT_r}{PC_r}\right)^{\sigma_r^{CTOT}} \cdot (b_r^C)^{\sigma_r^{CTOT}} \cdot (d_r^{CTOT})^{\sigma_r^{CTOT}-1} \cdot CTOT_r \tag{29}$$

$$CIM_r = \left(\frac{PCTOT_r}{PIM_r}\right)^{\sigma_r^{CTOT}} \cdot (b_r^{CIM})^{\sigma_r^{CTOT}} \cdot (d_r^{CTOT})^{\sigma_r^{CTOT}-1} \cdot CTOT_r \tag{30}$$

where PC_r and PIM_r are the domestic and import price indices. Finally, the shadow price of consumption ($PCTOT_r$) is given by the following modified optimum condition equation:

$$PCTOT_r \cdot CTOT_r = PC_r \cdot C_r + PIM_r \cdot CIM_r \tag{31}$$

Domestic composite consumption, on the other hand is another CES aggregate of different industrial products:

$$C_r = d_r^C \cdot \left[\sum_i b_{r,i}^{CR} \cdot \left(CR_{r,i}\right)^{\rho_r^C}\right]^{\frac{1}{\rho_r^C}} \tag{32}$$

where $CR_{r,i}$ is the consumption of households in region r of product i. $b_{r,i}^{CR}$ are the share parameters, d_r^C is the shift parameter and ρ_r^C is the substitution parameter. The consumption demand functions from this aggregator are as follows:

$$CR_{r,i} = \left(\frac{PC_r}{PCR_{r,i}}\right)^{\sigma_r^C} \cdot \left(b_{r,i}^{CR}\right)^{\sigma_r^C} \cdot (d_r^C)^{\sigma_r^C-1} \cdot C_r \tag{33}$$

The shadow price in this case is again determined by the concerning modified optimum condition:

$$PC_r \cdot C_r = \sum_i PCR_{r,i} \cdot CR_{r,i} \tag{34}$$

4.2.2. Investment demand

Total national investment in nominal terms is denoted by $INVNAT$. Total national investment is allocated to regional investment budgets according to fixed shares denoted by si_r:

$$PITOT_r \cdot ITOT_r = si_r \cdot \left(INVNAT + \sum_{r,i} (1 + tcomCIV_{r,i}) \cdot PD_{r,i} \cdot CIV_{r,i} - \sum_r PIM \cdot CIVM_r \right)$$

(35)

where $ITOT_r$ is total real investment in a region while $PITOT_r$ is its shadow price. This equation works only with the current setting of our model considering changes in inventories on the supply and not on the demand side. Each element in $CIV_{r,i}$ below zero means increase in inventories, and positive elements represents decrease (sales). Thus sales mean additional source for investment, while increased investment means a loss of potential investment (in the current period), these additional investments has to be produced and production must be financed by savings so it will decrease the amount of available investment funds. However imported inventory is treated differently. Positive *CIVM* means increase, negative means a decrease in inventories, so we need to subtract them from investment funds.

As with consumption, $ITOT_r$ is a composite of investment goods purchased from domestic sources and import according to a CES aggregator:

$$ITOT_r = d_r^{ITOT} \cdot \left[b_r^I \cdot (I_r)^{\rho_r^{ITOT}} + b_r^{IIM} \cdot (IIM_r)^{\rho_r^{ITOT}} \right]^{\frac{1}{\rho_r^{ITOT}}}$$

(36)

where I_r is investment from domestic sources, IIM_r is imported investment goods, d_r^{ITOT} is the shift parameter, b_r^I and b_r^{IIM} are the share parameters and ρ_r^{ITOT} is the substitution parameter. This aggregator gives rise to the following demand functions:

$$I_r = \left(\frac{PITOT_r}{PI_r} \right)^{\sigma_r^{ITOT}} \cdot (b_r^I)^{\sigma_r^{ITOT}} \cdot (d_r^{ITOT})^{\sigma_r^{ITOT}-1} \cdot ITOT_r$$

(37)

$$IIM_r = \left(\frac{PITOT_r}{PIM_r} \right)^{\sigma_r^{ITOT}} \cdot (b_r^{IIM})^{\sigma_r^{ITOT}} \cdot (d_r^{ITOT})^{\sigma_r^{ITOT}-1} \cdot ITOT_r$$

(38)

where PI_r and PIM_r are the domestic and import price indices. The shadow price of total regional investment is given by the following modified optimum condition:

$$PITOT_r \cdot ITOT_r = PI_r \cdot I_r + PIM_r \cdot IIM_r$$

(39)

Domestic investment, on the other hand is a CES aggregate of different industry products:

$$I_r = d_r^I \cdot \left[\sum_i b_{r,i}^{IR} \cdot \left(IR_{r,i} \right)^{\rho_r^I} \right]^{\frac{1}{\rho_r^I}}$$

(40)

where $IR_{r,i}$ is the investment in region r using product i. $b_{r,i}^{IR}$ are the share parameters, d_r^I is the shift parameter and ρ_r^I is the substitution parameter. The investment demand functions from this aggregator are as follows:

$$IR_{r,i} = \left(\frac{PI_r}{PIR_{r,i}}\right)^{\sigma_r^I} \cdot \left(b_{r,i}^{IR}\right)^{\sigma_r^I} \cdot (d_r^I)^{\sigma_r^I - 1} \cdot I_r$$

(41)

The corresponding shadow price (PI_r) is given by the following modified optimum condition:

$$PI_r \cdot I_r = \sum_i PIR_{r,i} \cdot IIR_{r,i}$$

(42)

4.2.3. Government demand

In case of government consumption, we follow a somewhat different approach. Government demand concerns almost exclusively services, on the other hand imports are usually materials, products rather than services which might suppose that there is no substitution between domestic and imported purchases. As a results (as opposed to other final demand components) we assume complementarity between domestic and imported goods and services. The same approach is followed in case of domestic industrial demand since assuming a cost-minimizing (optimizing) behavior in case of the government is not a usual assumption in CGE models. Most models assume fixed expenditure structure for the government or simply treat it exogenously (Lofgren et al, 2001).

First total nominal government expenditure (denoted by $GNAT$) is allocated to regions according to calibrated exogenous regional shares (sg_r):

$$PGTOT_r \cdot GTOT_r = sg_r \cdot GNAT$$

(43)

Where $GTOT_r$ is the total regional government consumption and $PGTOT_r$ is its price index. This total regional demand is than decomposed into domestic industrial and imported demand using the following Leontief function:

$$GTOT_r = min\left(\frac{1}{a_{r,1}^{GR}} \cdot GR_{r,1}, \frac{1}{a_{r,2}^{GR}} \cdot GR_{r,2}, \dots, \frac{1}{a_{r,i}^{GR}} \cdot GR_{r,i}, \frac{1}{a_r^{GIM}} \cdot GIM_r\right)$$

(44)

Where $GR_{r,i}$ is the domestic industrial government demand, GIM_r is the government demand for imported goods, on the other hand $a_{r,i}^{GR}$ and a_r^{GIM} are calibrated expenditure shares of these demand categories.

The demand functions from the Leontief function are the followings:

$$GR_{r,i} = a_{r,i}^{GR} \cdot GTOT_r$$

(45)

$$GIM_r = a_r^{GIM} \cdot GTOT_r$$

(46)

Finally, the shadow price ($PGTOT_r$) is determined by the following modified optimum condition:

$$PGTOT_r \cdot GTOT_r = \sum_i PGR_{r,i} \cdot GR_{r,i} + PIM_r \cdot GIM_r$$

(47)

Another problem in case of public services provided by the government is that there is no market for its services in reality. If the demand of a region is higher for these services than its supply, interregional trade (and price changes) should eliminate this difference. However since there is no market for these services, interregional trade should not exist in case of these categories. Since the SCGE model is based on an estimated interregional I-O table, this problem has to be treated in the estimation procedure. Although we treated this problem, our solution can be considered as partial since we allow for the interregional trade of government services but the magnitude of this trade is rather low compared to other activities. In Section A.4.3 the industry specific friction parameter is designed to capture industry specific attribute such as low level of tradability. As a result, there is smaller amount of interregional trade of government services. On the other hand, we could argue that this difference between the regional demand for and supply of government services might be caused also by interregional commuting which is not modelled in our approach but can influence our results. Furthermore, we could argue that some level of trade should exist since many government activates are concentrated in the capital city but the demand for these services is distributed in space across regions.

4.2.4. Export demand

Export demand for industry output is defined through two aggregated variables: real demand for exported goods towards the whole country ($EXNAT_j$) and its price index ($PEXNAT_j$). The real export demand is a CES composite of exported goods from different regions according to the following aggregator:

$$EXNAT_j = d_j^{EXNAT} \cdot \left[\sum_r b_{r,j}^{EXR} \cdot \left(EX_{r,j}\right)^{\rho_j^{EXNAT}}\right]^{\frac{1}{\rho_j^{EXNAT}}}$$

(48)

where $EX_{r,j}$ is the amount of industry product j bought on the export market from region r. $b_{r,j}^{EXR}$ are the share parameters, d_j^{EXNAT} is the shift parameter and ρ_j^{EXNAT} is the substitution parameter. This aggregator gives rise to the following demand functions for regional export:

$$EX_{r,j} = \left(\frac{PEXNAT_j}{PTEX_{r,j}}\right)^{\sigma_j^{EXNAT}} \cdot \left(b_{r,j}^{EXR}\right)^{\sigma_j^{EXNAT}} \cdot \left(d_j^{EXNAT}\right)^{\sigma_j^{EXNAT}-1} \cdot EXNAT_j$$

(49)

where σ_j^{EXNAT} is the substitution elasticity, linked to the substitution parameter: $\rho_j^{EXNAT} = (\sigma_j^{EXNAT} - 1)/\sigma_j^{EXNAT}$. As usual the shadow price ($PEXNAT_j$) is given by the following optimum condition:

$$PEXNAT_j \cdot EXNAT_j = \sum_r PTEX_{r,j} \cdot EX_{r,j}$$

(50)

4.3. Incomes and savings

In this section we briefly describe how income and saving is determined of households, the government and rest of world.

4.3.1. Households

Household income is defined at the regional level. Total income of regional households is made up of labor and capital incomes generated in regional industries:

$$YH_r = PLR_r \cdot LS_r + PKN \cdot KS_r$$

(51)

where YH_r is the total income of households. Households are assumed to save a constant fraction of their labor and capital income. In which we allow for the possibility of different saving rates in case of the two income types. Thus household saving (SH_r) is determined as follows:

$$SH_r = sL_r \cdot PLR_r \cdot LS_r + sK_r \cdot PKN \cdot KS_r$$

(52)

Where sL_r and sK_r stand for the saving rates of labor and capital income, respectively. Accordingly, households' consumption budget (BH_r) is defined simply as the difference between income and saving:

$$BH_r = YH_r - SH_r$$

(53)

4.3.2. Government

Government is assumed to have revenues from three sources: production tax, commodity (or product) tax and income tax (on factors). For all three taxes we define a region- and commodity/industry-specific exogenous tax-rate, then the revenue from production tax ($TAXPROD$) can be written as:

$$TAXPROD = \sum_r \sum_j tprod_{r,j} \cdot PVA_{r,j} \cdot VA_{r,j}$$

(54)

where $tprod_{r,j}$ are the tax rates in industries. Income tax revenue on factors is defined as:

$$TAXFAC = \sum_r \sum_j tlab_{r,j} \cdot PL_{r,j} \cdot L_{r,j} + \sum_r \sum_j tcap_{r,j} \cdot PK_{r,j} \cdot K_{r,j} \tag{55}$$

where $tlab_r$ and $tcap_r$ are the labor and capital income tax rates in industries. Finally, commodity tax revenues are defined as:

$$\begin{aligned} TAXCOM = & \sum_r \sum_i \sum_j \left(tcomXIR_{r,j} \cdot PQ_{r,i} \cdot a_{r,i,j} \cdot XR_{r,j}\right) \\ & + \sum_r \sum_i \left(tcomCR_{r,i} \cdot PQ_{r,i} \cdot CR_{r,i} + tcomIR_{r,i} \cdot PQ_{r,i} \cdot IR_{r,i} + tcomGR_{r,i} \cdot PQ_{r,i} \right. \\ & \left. \cdot GR_{r,i}\right) + \sum_r \sum_i tcomEX_{r,i} \cdot PEX_{r,i} \cdot EX_{r,i} - \sum_r \sum_i tcomCIV_{r,i} \cdot PD_{r,i} \cdot CIV_{r,i} \end{aligned} \tag{56}$$

Taxes on inventories are also added to tax revenues. Although inventory tax rates are negative we still need to account for them in government revenues (as certain expenditures, subsidies). Eventually these amounts will increase the total available investment fund in the economy. If tax rates would be positive, they would decrease total savings and increase government revenues.

After calculating tax revenues we can define government income as the sum of tax revenues:

$$YG = (TAXPROD + TAXFAC + TAXCOM) \tag{57}$$

Government expenditure is defined as the difference between tax revenues and government deficit (which is determined exogenously by Equation 82):

$$GNAT = YG - SG \tag{58}$$

We must note that this is only a partial approach to the government sector. Since the aim of this model is not to analyze the details of the government sector and due to poor data availability we decided not to account for the government sector in a comprehensive way. Most importantly we did not account for the transfer incomes and expenditures of government which means that our government deficit (which accounts for only a narrower aspect of the government) will differ from official data as a result of the differences in the way of calculation of them.

4.3.3. Foreign savings

Finally, the definition of foreign saving (current account balance in foreign currency) can be written as follows:

$$SW = \sum_r PWM_r \cdot \left(\sum_j \left(XIM_{r,j}\right) + CIM_r + IIM_r + GIM_r + CIVM_r \right) - \sum_j PWE_j \cdot EXNAT_j \tag{59}$$

Where *SW* refers to foreign savings in foreign currency and *er* refers to the exchange rate.

4.4. Supply of primary resources

In the current model version we allow for the mobility of both labor and capital between regions and industries as well. However this mobility is not perfect since we believe that perfect mobility would be an unrealistic assumption. As prices increase households will gradually allocate their resources accordingly which slowly decrease these price differences (ceteris paribus). In the next subchapters first we discuss labor mobility and then we describe the case of capital.

4.4.1. Supply of labor

In our model we assume that regional households own factors of production. They are allowed to decide where to allocate their scarce resources. In case of labor (as you can see it later) this adjustment (interregional migration) is more rigid and slower than in case of capital. We assume that the total national labor force is fixed in each period. Regional labor force is also fixed within each period however due to interregional migration we allow for some adjustment of the regional labor force with time. Migration choice is based on interregional differences in utility which consist real consumption and housing. As a result migration react to changes in utility with a one-year time lag. The complete description of migration is detailed in section 6.2. Since interregional mobility of labor is imperfect we assume some extent of imperfection in interindustry mobility too. We assume that households can offer their labor supply to different industries according to a CET function which means that transformation of labor from one sector to another is not perfect. Increasing wage differences will have less intensive reaction in labor supply. The CET function can be written as follows:

$$LS_r = d_r^{LR} \cdot \sum_i \left(b_{r,i}^{LI} \cdot {LI_{r,i}}^{\rho^{LI}} \right)^{\frac{1}{\rho^{LI}}}$$

(60)

where $LI_{r,j}$ is the industry level labor supply, d_r^{LR} is the shift parameter whereas $b_{r,i}^{LI}$ is the share parametersof CET function and ρ_r^{LI} is the transformation parameter. From which we can derive the industry level labor supply function:

$$LI_{r,i} = \left(\frac{PL_{r,i}}{PLR_r} \right)^{\sigma_r^{LI}} \cdot \left(b_{r,i}^{LI} \right)^{-\sigma_r^{LI}} \cdot \left(d_r^{LR} \right)^{-\sigma_r^{LI}-1} \cdot LS_r$$

(61)

The sectoral supply of regional labor ($LI_{r,i}$) has the same price as labor demand: $PL_{r,i}$, and PLR_r serves as the shadow price in the optimization. The total value of regional labor supply is formulated as follows:

$$\sum_i PL_{r,i} \cdot LI_{r,i} = PLR_r \cdot LS_r$$

(62)

4.4.1. Supply of capital

We assume that households can reallocate their capital resources between regions based on regional differences in capital prices.

The national capital stock is a simple aggregation of the regional stock owned by households:

$$KN = \sum_r KS_r \tag{63}$$

Where KS_r is the regional capital stock owned by households and *KN* is the national capital stock. The equation above implies that households are payed after their capital supply by national capital prices (PKN) which is selected as numeraire.

The national capital supply is then reallocated to regions based on a CET function as follows:

$$KN = d^{KN} \cdot \left(\sum_r b_r^{KR} \cdot {KR_r}^{\rho^{KR}}\right)^{\frac{1}{\rho^{KR}}} \tag{64}$$

Where KR_r is the regional capital supply, its supply function can be derived as follows:

$$KR_r = \left(\frac{PKR_r}{PKN}\right)^{\sigma^{KR}} \cdot (b_r^{KR})^{-\sigma^{KR}} \cdot (d^{KN})^{-\sigma^{KR}-1} \cdot KN \tag{65}$$

Where the price index of regional capital supply is PKR_r, the previously introduced *PKN* is in fact the shadow price of the optimization problem and d^{KR} is the shift parameter of the CET function and b_r^{KR} refers to the share parameter. The total value of national capital is given by the following equation:

$$\sum_r PKR_r \cdot KR_r = PKN \cdot KN \tag{66}$$

The idea behind this approach is that however capital can be allocated to any regions in the country, this allocation is not without some friction. The adjustment cannot react sensitively to regional price differences which can be determined by predetermining low elasticity values. Equation 66 is not included in the model since according to the Walras law one of the equations is considered to be redundant. In the epilogue of the model we only test if this equation is satisfied by the solution as a proof of Walras law.

In the next step this regional supply if further decomposed into industry-specific capital supply, without this step capital would be perfectly mobile between industries. For that purpose we employ another CET function:

$$KR_r = d_r^{KR} \cdot \sum_i \left(b_{r,i}^{KI} \cdot {KI_{r,i}}^{\rho^{KI}}\right)^{\frac{1}{\rho^{KI}}}$$

(67)

From which the industry level capital supply thus can be written as follows:

$$KI_{r,i} = \left(\frac{PK_{r,i}}{PKR_r}\right)^{\sigma_r^{KI}} \cdot \left(b_{r,i}^{KI}\right)^{-\sigma_r^{KI}} \cdot \left(d_r^{KR}\right)^{-\sigma_r^{KI}-1} \cdot KR_r$$

(68)

Where $KI_{r,i}$ is the sectoral supply of regional capital with the same price as capital demand: $PK_{r,i}$, and PKR_r serves as the shadow price in the optimization, d_r^{KR} is the shift parameter of industry level regional capital supply CET function and $b_{r,i}^{KI}$ refers to the share parameter of industrial capital supply. The total value of regional capital supply is formulated as follows:

$$\sum_i PK_{r,i} \cdot KI_{r,i} = PKR_r \cdot KR_r$$

(69)

4.5. Interregional trade and transportation

This is the final step of production and these composite products are used for consumption (final and intermediate). However, iceberg transportation costs are included in the prices of interregional trade prices which will bring spatial relationships in the prices. In this case the difference between FOB and CIF prices can be viewed as the transportation cost $(1 + \tau_{r,q,i})$. In this setting $\tau_{r,q,i}$ will determine the transportation cost between region *r* and *q* in case of an interregional import regarding industrial product *i*.

Total interregional trade is captured by $QR_{q,r,i}$ which shows us the total amount of goods and services transported between two regions. Later these products will be consumed/used by different actors in the destination region (e.g. households, government, etc.) The interregional trade is driven by the total amount of regional demand ($Q_{r,i}$ which will be discussed later) through a CES function. We do not explicitly model the interregional supply side. We assume that the regional demand will be satisfied from all of the regions through an equilibrium condition (see later Equation 78). The interregional CES function takes the following form:

$$Q_{r,i} = d_{r,i}^{Q} \cdot \left[\sum_q b_{q,r,i}^{QR} \cdot \left(\frac{QR_{q,r,i}}{(1 + \tau_{q,r,i})}\right)^{\rho_{r,i}^{Q}}\right]^{\frac{1}{\rho_{r,i}^{Q}}}$$

(70)

The interregional demand function is then derived (using the Lagrange method) which can be written as follows:

$$\frac{QR_{q,r,i}}{1+\tau_{q,r,i}} = \left(\frac{PQ_{r,i}}{(1+\tau_{q,r,i})\cdot PD_{q,i}}\right)^{\sigma_{r,i}^{Q}} \cdot \left(b_{q,r,i}^{QR}\right)^{\sigma_{r,i}^{Q}} \cdot \left(d_{r,i}^{Q}\right)^{\sigma_{r,i}^{Q}-1} \cdot Q_{r,i}$$

(71)

Finally, the total cost of regional demand must be equal to its interregional counterpart:

$$\sum_{q}(1+\tau_{q,r,i}) \cdot PD_{q,i} \cdot \frac{QR_{q,r,i}}{1+\tau_{q,r,i}} = PQ_{r,i} \cdot Q_{r,i}$$

(72)

4.5.1. Commodity tax on regional demand

Additionally commodity taxes are calculated on these sales. As a general case, we allow for specific commodity tax rates for all final users: e.g. households in region r may pay different tax rates on commodities than the tax rates paid by firms for instance. The general formulation allows for any kind of aggregation (e.g. we may simplify to have one single tax rate over the economy). The main reason for this formulation is that SAM data used for model calibration may supply commodity tax expenses separately for all final users and their spendings for different products. Let us denote the commodity tax rate of household spending, government spending, investment, industry intermediates, industry export respectively by $tcomCR_{r,i}$, $tcomGR_{r,i}$, $tcomIR_{r,i}$, $tcomXIR_{r,i}$, $tcomEX_{r,i}$, for purchases by users in region r. The respective post-tax prices are $PCR_{r,i}$, $PGR_{r,i}$, $PIR_{r,i}$, $PXIR_{r,i,j}$, $PTEX_{r,j}$. Then, the relationships between pre- and post-tax prices are given by the following relationships:

$$PCR_{r,i} = (1+tcomCR_{r,i}) \cdot PQ_{r,i}$$

(73)

$$PGR_{r,i} = (1+tcomGR_{r,i}) \cdot PQ_{r,i}$$

(74)

$$PIR_{r,i} = (1+tcomIR_{r,i}) \cdot PQ_{r,i}$$

(75)

$$PTEX_{r,j} = (1+tcomEX_{r,j}) \cdot PEX_{r,j}$$

(76)

As export demand is directly determined against the output of the different regions, export tax can be only interpreted separately for the source regions in the domestic economy.

4.6. Equilibrium conditions and model closure

The model is closed by the equilibrium conditions defined below.

First, equilibrium on the goods market is established on a region-region specific basis:

$$CR_{r,i} + IR_{r,i} + GR_{r,i} + \sum_{j} a_{r,i,j} \cdot XR_{r,j} = Q_{r,i}$$

(77)

Instead of using a CET function to distribute regional supply ($D_{r,i}$) to other destination regions we use another equilibrium condition which simplifies the model even more. This way we dropped our initial assumption of relational markets, in this formulation we assumed regional markets. This means also that the price index of $DT_{r,i}$ and $QR_{r,q,i}$ is the same $PD_{r,i}$ (FOB price).

$$DT_{r,i} = \sum\nolimits_{q} QR_{r,q,i}$$

(78)

Equilibrium condition for industry level capital market:

$$L_{r,i} = LI_{r,i}$$

(79)

Equilibrium condition for industry level capital market:

$$K_{r,i} = KI_{r,i}$$

(80)

Finally, total savings (saving of households, the government and the foreign sector) must be equal to investment:

$$INVNAT = \sum\nolimits_{r} SH_r + SW \cdot er + SG$$

(81)

Government saving is set by the deficit per GDP as a portion of current GDP.

$$SG = ed \cdot adj \cdot \sum_{r} \sum_{i} \left(PVA_{r,i} \cdot VA_{r,i} \right)$$

(82)

Where the value of *ed* (deficit per GDP) is updated in each time period by the macro block according to a restrictive deficit rule (discussed in a later chapter).

With respect to exports, we have the choice to fix either export price level or export quantity. Once one of these is fixed, Equation 50 defines the other. We decided to leave quantities to be determined and set an exogenous world export price and determine the export price in home currency according to the following equation which is a standard approach in CGE and SCGE models (see Lofgren et al, 2001, Mercenier et al., 2016):

$$PEXNAT_j = PWE \cdot ER$$

(83)

where PWE is a parameter fixing world export prices.

Import prices must also be fixed without the specification of a foreign supply scheme:

$$PIM_r = PWM \cdot ER \tag{84}$$

Where PWM is the exogenously fixed world price index of import.

5. The MACRO block

The purpose of macro block is to relax the most important macroeconomic closures of the model. Initially government deficit was set to a known initial value (SG0). Now the macro block will update its annual value according to the naive rule set by Hungarian laws for the path of government debt.

The accumulation of government debt per GDP is controlled by the following equation:

$$b_t = b_{t-1} + b_{t-1} \cdot (i - \pi - g) + ed_t \tag{85}$$

Where b_t and b_{t-1} are the government debt (per GDP) in time period t and t-1, i represents nominal interest rate, π is the inflation rate and g is the national growth rate of GDP.

The rule for controlling deficit (and national debt) can be described as follows:

$$ed_t - \overline{ed} = -\mu \cdot (b_{t-1} - \overline{b}) \tag{86}$$

Where ed_t is the actual deficit ratio in time period t, $\overline{ed}$ is the target deficit ratio (calibrated from available data), $\overline{b}$ is the target deficit (set to 50%) and *β* is the sensitivity of adjustment (exogenously fixed in order to provide a smooth path for deficit and debt). The problem with this kind of approach is that the sensitivity of deficit is too strong in the first couple of periods. Thus, we used a slightly modified version of the model. We adjusted the value of β in all time periods accordingly. In the first couple of periods to slow down the adjustment mechanism we set a small positive value which made it possible to reach a smaller deficit than before. Then we start to increase β annually and as we move on with time we set stricter deficit rule.

$$ed_t = \overline{ed} - \mu_{t-1} \cdot (b_{t-1} - \overline{b}) \tag{87}$$

With an initial positive value of μ government deficit would have been set suddenly from a 3% deficit to a 2% surplus which we found unrealistic thus we slowed down this process by applying an initial negative value for sensitivity. Negative μ means that deficit can increase (or at least be higher than the target deficit level) even if debt per GDP is above the target level but because this value is small actual deficit will be less and less than before and with the continuous update of β there will be a smooth convergence to target deficit and debt. The starting value of debt (b_{2010}) is 74.37% and the value of deficit (ed_{2010}) is 3.10%.

Actual equations in the model:

$$ed_{t+1} = \overline{ed} - \mu_t \cdot \left(b_t - \overline{b}\right)$$

(88)

$$b_{t+1} = b_t + b_t \cdot (i - \pi_t - g_t) + ed_{t+1}$$

(89)

Where π_t and g_t is calculated from the SCGE model in each time period. The actual level of the deficit is set by a new equation (which can endogenize variable SG):

$$SG_t = ed_t \cdot adj \cdot \sum_r \sum_i \left(PVA_{r,i} \cdot VA_{r,i}\right)$$

(90)

Where parameter *adj* is responsible to provide consistency between model results and official data. Since the government expenditure and budget calculated from the input-output table does not contain all elements thus deficit will differ from official data. The reason for such differences can be found in the modelling approach. Industries like education have a huge public involvement and yet these industries are still considered to operate as a private industry in the model. Since input-output tables are published in a way that organization (either private or public) are classified into the corresponding industry based on their primary activity. Thus, the consideration of total expenditure and income of the government (within the IO table) does not reflect the official data. Thus, *adj* parameter is calculated to reflect differences between actual deficit and the model based one (SG0):

$$adj = {ed_{2010} \cdot GDP_{2010}}/{SG0}$$

(91)

6. Model dynamics

The dynamic behavior of the model is introduced in four different steps. First, we describe the process of capital accumulation then we introduce the interregional migration of labor. Third, we show the way to account for productivity changes at the regional level. Fourth, we describe the determination of government deficit based on a naïve deficit rule. These four dynamic aspects of the model are 'separated' from the model itself. They are partially determined outside the model after each time period. Since the model is recursive, each period can be considered as a static model and these static steps are connected through by these for different processes.

6.1. Capital accumulation

The accumulation of capital is determined by a standard cumulative equation:

$$KS_{r,t+1} = (1 - \delta) \cdot KS_{r,t} + \frac{PITOT_{r,t} \cdot ITOT_{r,t}}{PKR_{r,t}}$$

(92)

Where $KS_{r,t}$ is the capital supply in region r and time period t, δ is the depreciation rate. The capital supply has a role only in the regional income determination because we assumed previously that we have one national capital market.

6.2. Migration

We assume that only interregional migration determines the change in regional labor supply since we ignore demographic changes, international migration and unemployment. Although we are aware that all of them plays an important role in the determination of the growth potential of Hungary. We decided however to treat them exogenously (in the current setup) mainly due to lack of data.

The determination of migration is based on the tradition of the GMR modelling (Varga, Sebestyén, Szabó, Szerb 2018). Interregional net migration is accounted for in the model which is determined by the following equation based on the difference between regional ($U_{r,t}$) and the national average utility per capita:

$$LMIGR_{r,t} = LS_{r,t} \cdot \varphi \cdot \left(e^{\theta \cdot U_{r,t}} - e^{\theta \cdot \frac{\sum r\, U_{r,t} \cdot LS_{r,t}}{\sum \mathrm{r}\, LS_{r,t}}} \right)$$

(93)

Where $LMIGR_{r,t}$ stands for the net immigration in region r in terms of employees (millions of HUF in the model since wages are set to unity), $LS_{r,t}$ denotes regional labor supply. On the other hand, φ and θ are sensitivity parameters and their values are borrowed from a previous version of GMR-Hungary (Járosi et al., 2009). This approach suggests that regions that can offer lower utility level than the national average might face labor (and population) out-migration and vice versa. The regional utility function is based on total regional consumption ($CTOT_r$) per capita and regional housing (H_r) per capita (in which total housing is treated exogenously in the current model setup):

$$U_{r,t} = initC_r + \alpha H \cdot \ln\left(\frac{H_{r,t}}{N_{r,t}}\right) + \beta H \cdot \ln\left(\frac{CTOT_{r,t}}{N_{r,t}}\right)$$

(94)

Where $initC_r$, αH and βH are parameters that are calibrated, $CTOT_{r,t}$ is the total regional consumption and $N_{r,t}$ is the regional population. The values of αH and βH are borrowed from the original GMR-Hungary model, $initC_r$ is calibrated in in order to achieve complete consistency between calculated net population migration ($NMIGR_{r,'2010'}$ - to be later introduced) and actual net domestic migration data in 2010. Next we can determine the change of regional labor supply (stock) as follows:

$$LS_{r,t+1} = LS_{r,t} + LMIGR_{r,t}$$

(95)

After that we need to update the regional population. We assume that as labor supply is changed by migration, population is changed accordingly. The equation is based on the difference between net regional migration and the average migration multiplied by the fixed conversion ratio between labor and population (calibrated in the base year):

$$NMIGR_{r,t} = \left(LMIGR_{r,t} - LS_{r,t} \cdot \frac{\sum_r LMIGR_{r,t}}{\sum_r LS_{r,t}}\right) \cdot \frac{\sum_r N_{r,t}}{\sum_r LS_{r,t}}$$

(96)

Finally, the change of population can be described in a similar way to labor migration:

$$N_{r,t+1} = N_{r,t} + NMIGR_{r,t}$$

(97)

Where $NMIGR_{r,t}$ stands for the net migration of population (number of people).

6.3. Changes in TFP

The SCGE block provides input for the TFP block between each time period. As regional labor supply changes (LS_r) productivity is changed according to the TFP equation. This change of productivity is then channeled back to the production function of the SCGE model:

$$aCD_{r,i,t+1} = \frac{TFP_{r,t+1}}{TFP_{r,t}} \cdot aCD_{r,i,t}$$

(98)

In the current setup due to lack of detailed industry level data we assume that industrial TFP change proportional to its regional counterpart. Thus, TFP growth changes the shift parameter ($aCD_{r,i,t+1}$) of regional industry level value added CD production function (see Equation 8) in the next time period. In its simplest form, we assume that regional TFP growth will affect all industries in the region uniformly due to lack of regional industry-specific data.

Since the database for the two blocks are fundamentally different complete consistency between the two blocks cannot be achieved. However, we try to harmonize the two most important variables that are commonly used by them: TFP and labor. The basic difference is that to calibrate the regional TFP values we used regional GDP, employment and (estimated) capital stock values. However, in the SCGE model we only have value added as the output of industries which differs from GDP. Another issue is that the values in the TFP block are calculated in base year prices (2000). As a result, we chose to adopt the calibrated TFP values and labor inputs from the TFP sub-model and recalibrate other variables in the SCGE block (including wages, price of capital and capital stock).

The idea behind the recalibration is that we define a regional production function for each region (since there is no industrial dimension in the TFP block):

$$Y_r = TFP_r \cdot L_r^{\alpha_r} \cdot K_r^{\beta_r}$$

(99)

Where TFP_r and L_r are the adopted TFP and employment values from the TFP block. The share parameters can be calibrated based on the interregional I-O table as follows:

$$\alpha_r = \frac{\sum_i SAM_{r,i}^{LAB}}{\sum_i SAM_{r,i}^{LAB} + SAM_{r,i}^{CAP}}$$

(100)

$$\beta_r = 1 - \alpha_r$$

(101)

Where $SAM_{r,i}^{LAB}$ and $SAM_{r,i}^{CAP}$are the total industrial expenditures on labor and capital inputs respectively. The total regional value added is given again by the I-O tables.

$$Y_r = \sum_i SAM_{r,i}^{LAB} + SAM_{r,i}^{CAP}$$

(102)

Where we assume that the regional price index of value added is normalized to unity. At this point all factors of the production function is determined except the regional capital stock which is calculated residually:

$$K_r = \left(\frac{Y_r}{TFP_r \cdot L_r^{\alpha_r}}\right)^{\frac{1}{\beta_r}}$$

(103)

Finally, factor prices are given as follows

$$PLR0_r = \frac{\sum_i SAM_{r,i}^{LAB}}{L0_r}$$

(104)

$$PKR0_r = \frac{\sum_i SAM_{r,i}^{CAP}}{K0_r}$$

(105)

Since industry level employment is given we are able to calculate industry-specific wages:

$$PL0_{r,i} = \frac{SAM_{r,i}^{LAB}}{L0_{r,i}}$$

(106)

In case of capital due to lack of data we assume that initial industry-specific capital prices are identical to their regional counterparts:

$$PK_{r,i} = PKR_r$$

(107)

Finally, we can calibrate the industry-specific production functions. In order to do that first we calibrate the initial variables based on the I-O table:

$$VA0_{r,i} = SAM_{r,i}^{LAB} + SAM_{r,i}^{CAP}$$

(108)

$$K0_{r,i} = \frac{SAM_{r,i}^{CAP}}{PK0_{r,i}}$$

(109)

Since industry-specific employment is given in the data we do not need to calibrate it. The industry-specific share parameters are calibrated in the same manner as before:

$$\alpha_{r,i} = \frac{SAM_{r,i}^{LAB}}{SAM_{r,i}^{LAB} + SAM_{r,i}^{CAP}}$$

(110)

$$\beta_{r,i} = 1 - \alpha_{r,i}$$

(111)

Finally, the industry specific TFP values can be derived from the production function:

$$TFP_{r,i} = \frac{VA0_{r,i}}{L0_{r,i}^{\alpha_{r,i}} \cdot K0_{r,i}^{\beta_{r,i}}}$$

(112)

As a result, we integrated the most important variables of the TFP block and we managed to maintain consistency between the model and the estimated I-O table.

7. Sensitivity analysis of the SCGE model

In this section we provide a brief description of the robustness check of the SCGE model. We change the elasticities of all CES and CET functions to test weather changes in these values have a significant impact on modelling outputs. We change these elasticities by +/-5% and we analyze the effects of all the possible combination of changes in case of these elasticities. In the model we have 11 different elasticity values thus increasing and decreasing them would result in 2^{12} = 4096 different simulations. We measure the robustness by the percentage changes of regional-industrial value added during the whole simulation period (2010-2029). This means that altogether 4096*20*37 = 3 031 040 different relative deviations were accounted for in the sensitivity analysis. If the modelling results are robust the majority of these impacts are concentrated around zero.

To illustrate our finding we created a histogram depicting the distribution of effects. Based on these we can state that 5% changes in elasticities causes less than 1% deviation in industry level GVA which can be considered as a robust modelling setup. It is still important to note that in case of an elasticity with the value of 1.5% +/-5% of deviation would result in 1.425% and 1.575% which deviation can be still considered as not too large in comparison to what can be found in the literature for some elasticities.

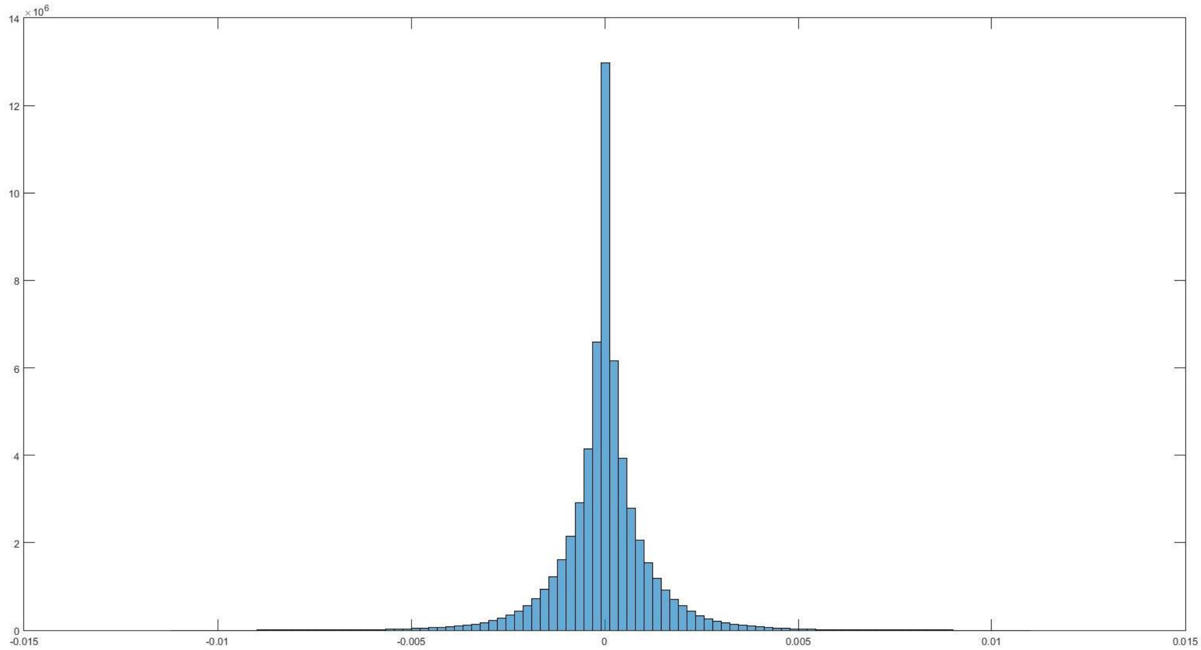

Figure 10: Sensitivity analysis histogram

In Figure 10 the histogram is based on these relative changes of regional industry level GVA compared to the baseline case (when all the elasticities are unchanged) which illustrates the distribution of deviation from the baseline paths. Note that in this SSA we did not changed the elasticity of substitution of the value added production function. The primary reason for that according to the international literature is that the elasticity of substitution between capital and labor does not differ significantly from unity (Cobb-Douglass function). Furthermore, since this Cobb-Douglas production function was the basis of the estimation of regional TFPs we did not want to deviate from this functional form.

8. Sample policy impact simulations

8.1. GDP impacts of TFP simulations

In this section we briefly describe some illustrative simulation results. We calculate the impacts of two types of policy shocks on regional TFP and GDP values. Four policy variables are in target: R&D expenditures, ENQ-index, human capital and the REDI-index. In the first series of shocks we increased the regional values of these four variables by the 1 % of their respective regional average values between 2015 and 2019. In the second series of shocks the policy variables are increased by the 1 % of their national averages for the same time period.

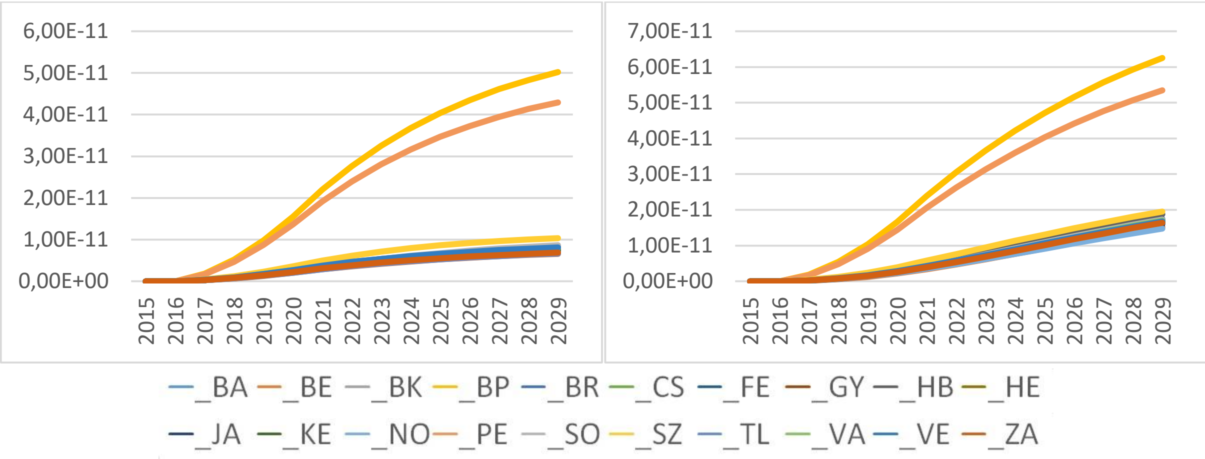

Figure 11: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% regional ENQ shocks

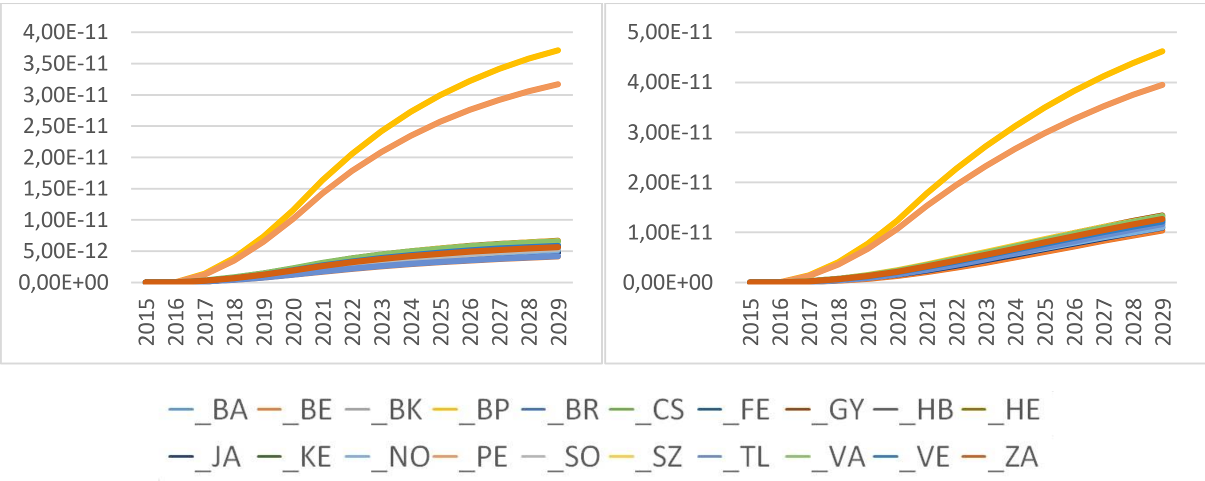

Figure 12: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% national ENQ shocks

We start with the ENQ simulations. ENQ influences knowledge production directly (in the patent equation) with a one-year time lag. The changes in patent stock influence productivity with another time lag thus both productivity and GDP impacts become visible in 2017. The highest impacts are expected in Budapest and Pest county since ENQ interacts with R&D support in the TFP equation which is very high in Budapest and Pest county. Additionally, due to agglomeration effects Budapest and Pest are more strongly influenced by the positive effects of the spillovers of regional patent stock accumulation. All the other regions are affected in a similar way with much lower growth paths. The TFP and GDP paths follow very similar patterns, however GDP paths seem to be steeper as a result of additional cumulative effects (e.g. additional investment as an indirect effect of the policy shock). In case of the national shock we can observe similar patterns.

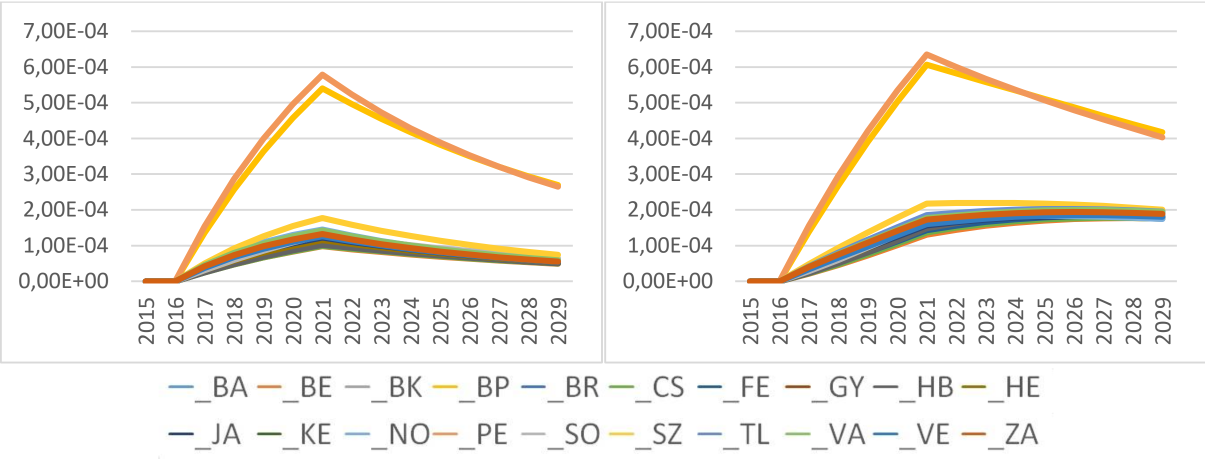

Figure 13: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% regional R&D support

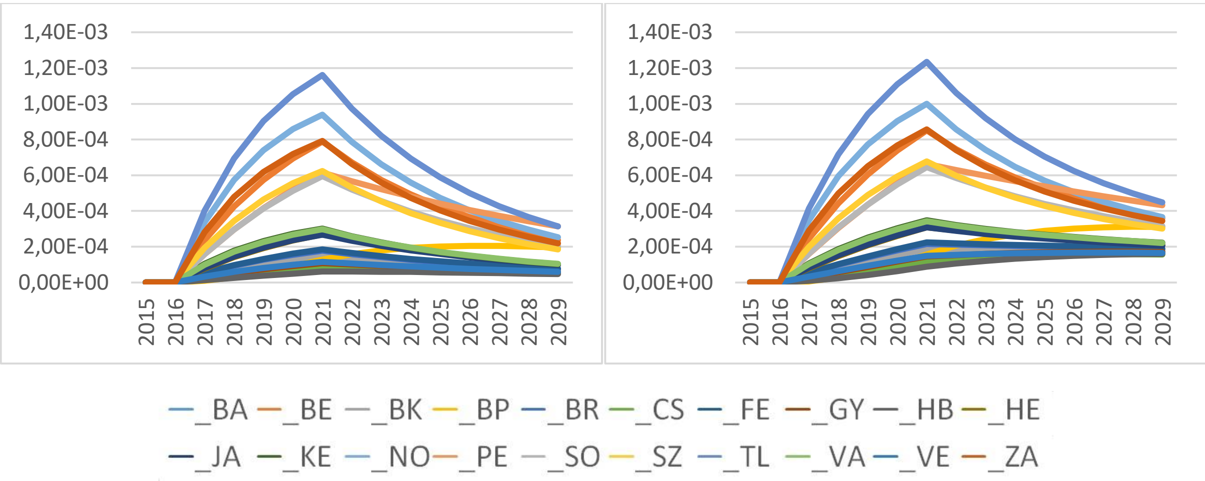

Figure 14: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% national R&D support

R&D support enters the patent equation of the TFP block and it interacts with the ENQ. Thus, the effects of the regional shocks are higher in regions where R&D support was initially higher, where network connections are stronger (high ENQ) and in regions which better utilize patent accumulation. As a result, Budapest and Pest gained the most from the interventions. Since R&D support is a temporary shock it runs out at the end of the five-year period and the further positive effects are continuously decreasing as patent accumulation is depreciated. In case of the national shocks we can see that regions with small initial R&D supports gain the most because in their case the shocks are large compared to their general R&D expenditures. In both cases TFP effects mainly drive GDP impacts the small differences between GDP and TFP impacts can be attributed to similar effects as before.

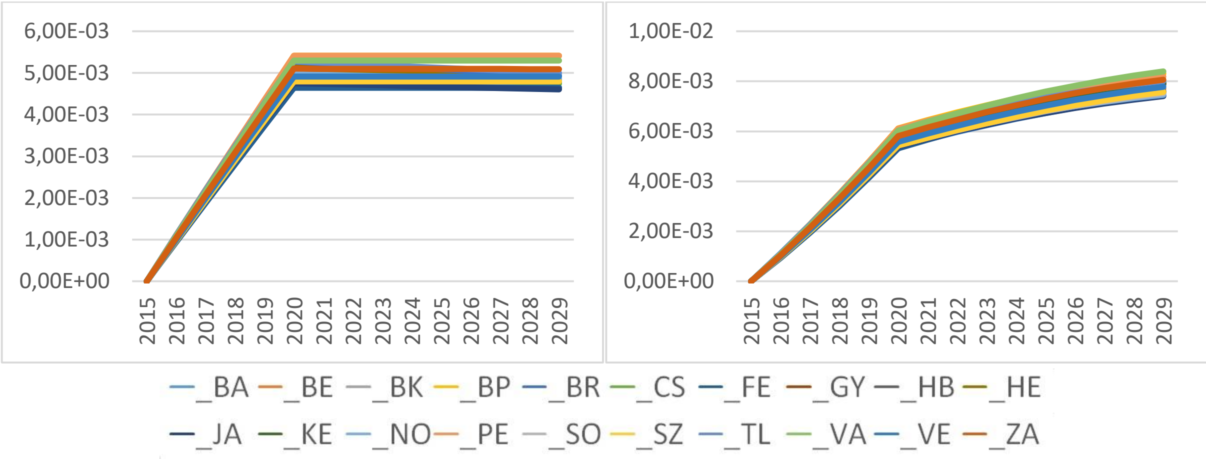

Figure 15: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% regional human capital shock

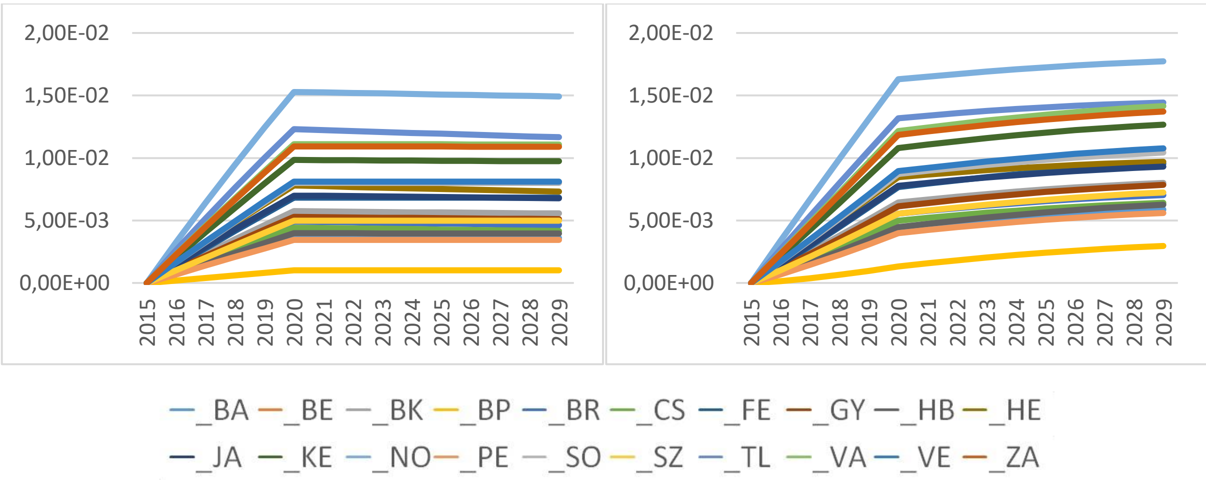

Figure 16: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% national human capital shock

For the case of human capital, we run the same simulations. Human capital influences productivity directly with only a one-year time lag thus the effects are already visible in 2016. This also means that patent accumulation is not affected directly by the shock thus additional productivity growth is not generated by the accumulation of patents. Human capital interacts with REDI in the TFP equation assuming that human capital is better utilized if the region is entrepreneurially more developed. As a result (although the effects stay very close to each other) entrepreneurially developed regions perform slightly better than others (Budapest, Pest, Zala, Vas, Tolna) in case of the regional shock. For human capital and REDI shocks further dynamic elements such as labor and capital migration also influence the growth paths. The positive slopes of the GDP paths continue after the interventions which is the result of additional capital accumulation resulting from higher productivity and income generated by the shock. In case of the national shock there is a larger variation in the growth paths since the relative size of the shock compared to the initial regional human capital stock is much larger for small and less developed regions.

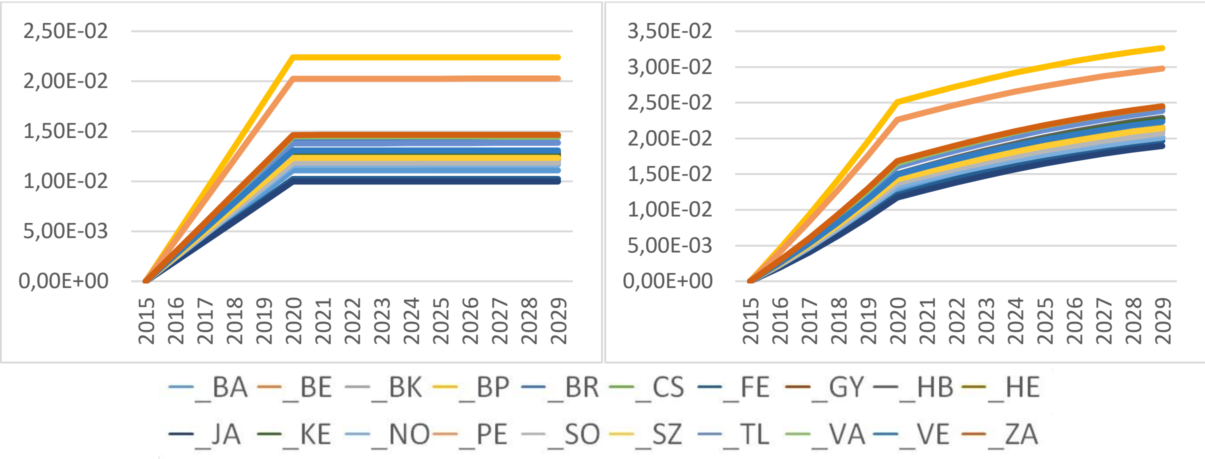

Figure 17: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% regional REDI shock

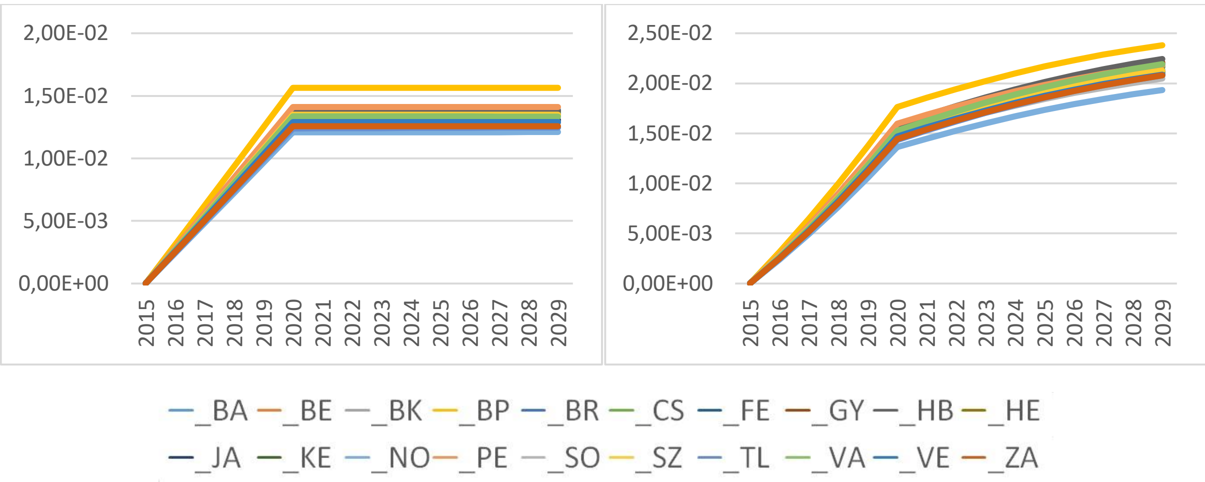

Figure 18: The TFP (left-hand panel) and GDP (right-hand panel) impact of 1% national REDI shock

REDI interacts with the human capital stock in the TFP equation. As a result, regions with large human capital stock better utilize the same REDI shock in their productivity increase. In case of regional shocks, those regions that have more developed entrepreneurial ecosystems (high initial REDI) benefit the most since their absolute REDI change is larger. Entrepreneurially more developed regions (with high human capital as well) such as Budapest, Pest and Zala gain the most from the regional shocks. While lagging regions with low initial REDI or human capital stocks gain the least productivity and GDP effects (e.g. Jász-Nagykun-Szolnok, Nógrád). In case of the national shock, regions follow a very similar paths. Effects are predominantly determined by regional differences in the human capital stocks. Thus Budapest, Pest, Győr-Moson-Sopron and Hajdú Bihar utilize the REDI shocks the best. On the other hand lagging regions with low human capital stock (e.g. Nógrád, Somogy, Tolna) performs the worst in this scenario. Long run paths are also influenced by migration and accumulation of capital (investment) and labor.

8.2. Regional GDP impacts of industry-specific investment support

In this section we apply our framework to analyze the economic effects of industry-specific investment support. The aim of these simulations is to illustrate the capabilities of our modeling framework, so we restrict the analysis to a limited set of regions. The sample simulations are carried out for three Hungarian NUTS 3 regions with significantly different economic potentials but still representing typical Hungarian counties: Budapest, Győr-Moson-Sopron and Baranya counties. Budapest is the most developed region of Hungary, whereas Győr-Moson-Sopron is a traditional industrial region and Baranya is a rural county.

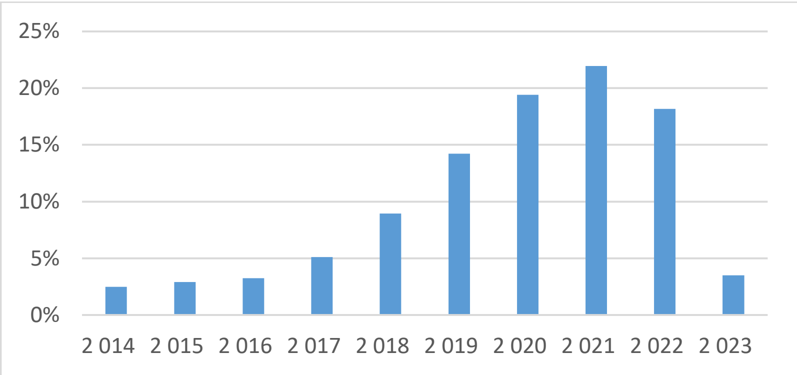

Figure 19: The distribution of investment support over time

In our scenarios, we examine the effects of an identical, but separate investment support to every industry in a region. We set the size of the industry-specific intervention equal to 1% of the total regional capital stock. Then we distribute this investment support over 9 years between 2014 and 2023 based on the expected trend of the distribution of EU funds (illustrated by Figure 19) which is based on past Hungarian experiences. Economic impacts are measured as the average annual change of total regional value added between 2014 and 2029. Therefore, in this experiment the interest lies in total regional effects of industry support policies.

In our illustrative simulations we highlight the basic drivers of regional economic growth in case of an identical industry specific investment support. The economic effects are the result of complex interactions of different mechanisms in the impact assessment model. In what follow we try to elaborate the most important determinants of potential regional growth possibilities.

First, we found in our research that capital intensity of industries is highly important in the determination of potential regional growth. In the capital, Budapest, capital intensive industries tend to perform better at promoting higher growth (ceteris paribus), on the contrary labor-intensive industries tend to perform better in other regions.

In the baseline simulation (without any interventions) most of the regions (except Budapest and some industrialized counties) have to face negative net migration. As a result of the investment support (and the capital created as a result) regions gain more capability to keep or even to attract more labor via migration. This effect is stronger if regional wages increase more and if regional labor demand increase more. Labor demand can be increased stronger by labor intensive industries since their technology prescribes that using the same amount of labor labor-intensive industries can produce more than others (ceteris paribus). Thus, supporting labor-intensive industries regions become more able to increase their attraction in the interregional labor market. These effects are further fostered by negative agglomeration externalities in the countryside since these effects are much weaker there as opposed to the capital.

In Budapest however we experience a great amount of net immigration in the baseline simulation. This continuous immigration increases the effects of negative agglomeration externalities. Thus, supporting labor intensive activities in Budapest can be less efficient since negative agglomeration effects might be further strengthened and as a result it becomes relatively harder to attract even more labor to the capital. Thus, in the capital we observed that capital intensive activities perform better at creating higher regional growth.

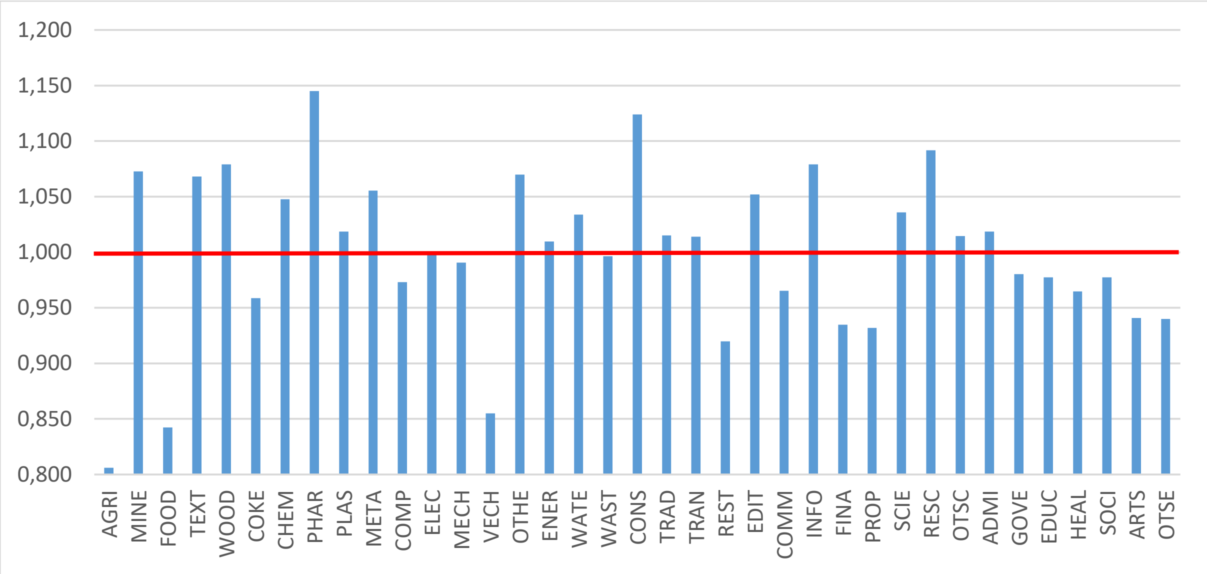

Figure 20: Regional GDP impacts of different industry-specific investment support in Budapest

Thus, the industry-specific support of labor-intensive activities creates larger abundance of labor in the countryside than in the capital. As a consequence, these industries are capable of creating higher growth in regions outside Budapest, however, capital intensive activities are better capable of increasing local growth in Budapest.

The role of capital/labor intensity is crucial in explaining the effects of industry-specific investment support. However, this factor is not the only determining factor that influences economic effects. Strong input-output linkages both at the purchase (backward linkages) and sale (forward linkages) side are important in stimulating other industries indirectly. Industries that are highly embedded in the local economy are better capable of creating additional local growth through a local multiplier effect.

Furthermore, we found that it is not enough if an activity has strong interindustry linkages. Backward and forward linkages determine how total regional output should be changed as a result of an additional unit of output produced by an activity in the region. Since we measure economic effects by regional value added it is extremely important to know how productive the local activities are. This factor is measured by our calibrated industry-specific total factor productivity (TFP) values. If a selected industry has strong interindustry linkages positive regional economic effects can be further amplified if these linkages are connected to sectors that are characterized by high productivity (TFP) since sectors with higher TFP can produce more using the same amount of inputs. As a result, if productive industries are supported indirectly, they can attract more factors of production (partially from less productive sectors and/or regions) and as a consequence regional production can be further enhanced.

These effects are further complicated by foreign relations. While as a result of potentially decreasing prices increasing foreign demand can serve as another source of local growth, however industries that are highly dependent on foreign inputs have less capabilities to positively affect regional production since import expenditures weaken the local multiplier effect.

Finally, additional income created as the interplay of all the above-mentioned effects can further increase the production of some sectors that satisfy different groups of final demand groups (consumption, investment, government).

In what follows we provide some insights into the economic impacts of industry-specific investment support in our selected regions: Budapest, Győr-Moson-Sopron and Baranya. In Figure 20 we can see the economic effects of supporting activities in Budapest. As we mentioned before in Budapest on average capital intensive activities are better capable of promoting higher local growth. However, given the complex mechanism that eventually determines growth even slightly labor-intensive sectors can create large local effects. Our results suggest that mostly service activities (informatics, scientific activities, R&D, editing and publishing) and some highly productive industries (pharmaceutical) and some other manufacturing industries perform the best in terms of stimulating the local economy.

For example, pharmaceutical industry in Budapest is one of the capital intensive sectors, it has relatively weak local inter-industry linkages, but it can be categorized as a basic industry since it is heavily export oriented, and on the other hand import expenditures are less significant. As a result, pharmaceutical activities are capable of creating large local growth. Research and development is similarly slightly capital-intensive, however it has also relatively weak inter-industry linkages but these linkages (especially backward linkages) are connected to actors that are highly productive. On the contrary construction is also one of the top activities in Budapest which can be characterized with a slightly labor-intensive technology. Although labor intensive activities in general are less capable of creating growth in the capital if other factors of success are also present even this alternative can be a good specialization. Construction has strong backward linkages in the local economy, although its key partners are not considered as highly productive. However, more than 50% of local investment demand is focused on construction services. Due to additional investment support, and its positive income effect, savings and investments are increased even after interventions run out. As a result of additional local investments and its demand towards construction, supporting construction can be a good alternative in Budapest. Information services are also characterized as labor-intensive activity, however its strong I-O linkages are connected to highly productive local actors which makes high local growth possible.

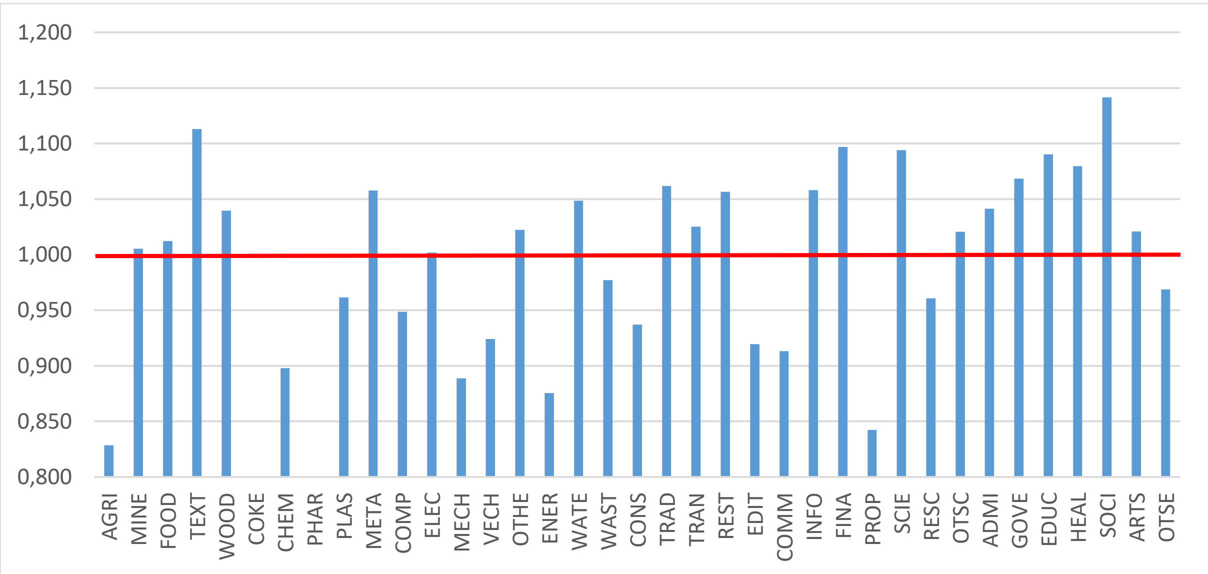

Figure 21: Regional GDP impact of different industry-specific investment support in Győr-Moson-Sopron

In Győr-Moson-Sopron we can find some similarities and some significant differences (Figure 21). First, many labor-intensive industries (social care, financial services, scientific activities, government related activities, etc.) can stimulate significantly local economic performance. Interestingly the textile industry (which is labor-intensive) is both successful in Budapest and in the countryside as well. This can be explained by the database, an interregional input-output table we used for calibration. Textile industry is characterized by high export orientation while its import expenditures are significantly lower. As a result, additional foreign expenditure (demand) can remain within the region creating further positive indirect effects. Győr-Moson-Sopron is an industrialized county, dominated by the manufacture of vehicles. Our results suggest that although it is a significant industry in the regions its local I-O linkages are weak, most of its inputs are imported as a result the local multiplier effect is limited. Other related industries (such as manufacture of metallic products) might perform better due to their stronger local linkages.

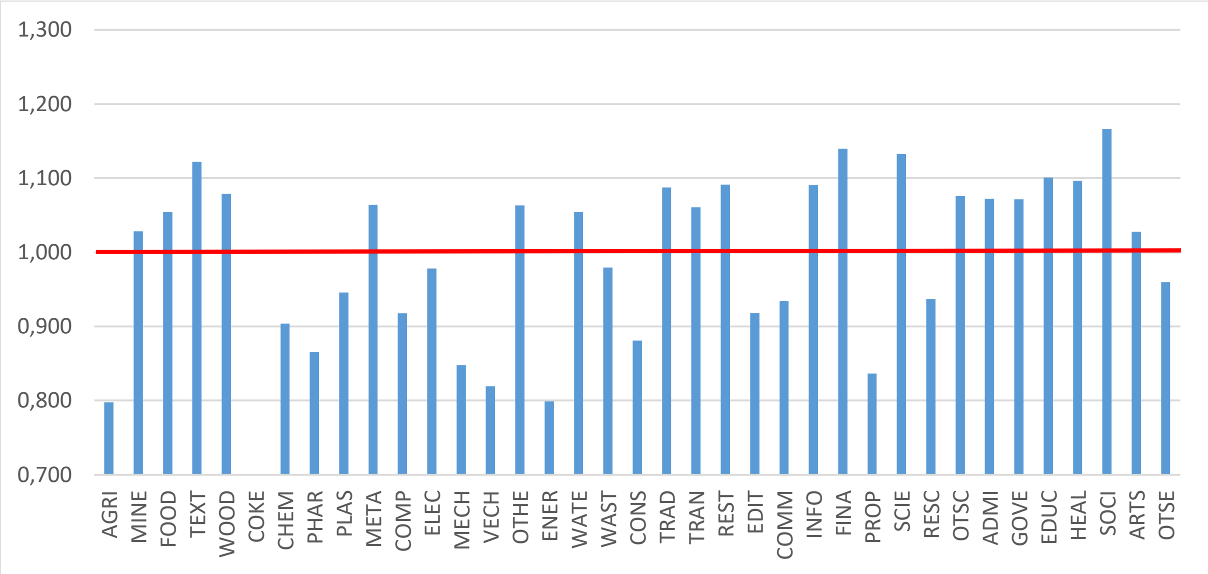

Figure 22: Regional GDP impact of different industry-specific investment support in Baranya

Baranya (Figure 22) shows a lot of similarities with Győr-Moson-Sopron since labor intensive activities are considered to be better in stimulating the local economy. Although economic effects can be somewhat different. For example, the manufacture of food and beverages perform clearly better in Baranya due to its higher embeddedness in the local economy. Similar argument can be given in case of a number of sectors (e.g. food and paper production, other manufacturing, accommodation and restaurants, scientific activities, etc.). On the other hand, most manufacturing sectors (e.g. manufacture of computers, electric equipment, machinery, vehicles) are more competitive in Győr-Moson-Sopron.

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Appendix

A.1. The list of parameters and model variables

Sets

i,j	-	Industries (37 aggregated NACE 2 industries, for detailed description see A.2)
k	-	Commodities (65 CPA commodity groups)
r,q	-	Regions (19 NUTS III counties and Budapest)
t	-	time periods (2010-2029)

The regionalization method

x_i^r	-	Regional gross output by industries
x_i^N	-	National gross output by industries
ε_i^r	-	Regionalizing factor (the share of regional employment in industry *i* compared to its national counterpart)
Emp_i^r	-	Regional industrial number of employees
Emp_i^N	-	National industrial number of employees
$V_{i,k}^r$	-	Regionalized supply table (*i* x *c*) /without import row/
$V_{i,k}^N$	-	National supply table (*i* x *c*) /without import row/
$U_{k,i}^r$	-	Regionalized use table (*c* x *i*) /without final demand and value added block/
$U_{k,i}^N$	-	National use table (*c* x *i*) /without final demand and value added block/
q_k^r	-	The regional total supply of commodity *c*
q_k^N	-	The national total supply of commodity *c*
IMP_k^r	-	Regional import of foreign commodities
IMP_k^N	-	National import of foreign commodities
$EXP\_GR_k^R$	-	Regional gross export of domestic commodities
$IMP\_GR_k^R$	-	Regional gross import of domestic commodities
EXP_k^r	-	Regional foreign export of commodities
EXP_k^N	-	National foreign export of commodities
IM_k^r	-	Regional net import of domestic commodities
IM_k^N	-	National net import of domestic commodities
VA_i^r	-	Regional industrial value added
VA_i^N	-	National industrial value added
TAX_i^r	-	Regional commodity taxes and subsidies
TAX_i^N	-	National commodity taxes and subsidies
FD_{km}^r	-	Regional commodity purchase of final users *f*
FD_{km}^N	-	National commodity purchase of final users *f*
RIM_c^R	-	Net regional import of domestic commodities
CH_i	-	Cross-hauling ratio of commodity *c*
ω_i	-	Product heterogeneity
EX_i^r	-	National export of commodity *c*
$D_{i,c}^r$	-	Regional market share matrix
$B_{c,i}^r$	-	Regional absorption matrix
$A_{i,j}^r$	-	Regional input-output coefficient matrix (derived from the absorption and market share matrices)
$A\_FD_{j,m}^r$	-	Regional coefficient matrix of final users
$Z_{i,j}^R$	-	Interindustry transaction matrix
$Z\_FD_{i,m}^R$	-	Final use matrix

$T_i^{r,q}$	-	Interregional (origin-destination) trade flows (gravity method)
s_i^r	-	Total domestic product outflow (shipment) of industry *i* in region *r*
K_i^q	-	Total domestic demand for industry i's product in region *q*
$F_i^{r,q}$	-	Friction factor (inverted measure of distance) between region *r* and *q*
λ^i	-	The sensitivity of trade to distance (for industry *i*)
$d_{r,q}$	-	The geographical distance between region *r* and *q*
LM_i	-	"local market": the share of domestic demand compared to the total demand of industry *i*
CP_i	-	the share of the largest exporting region within the total shipment of industry *i*'s product (the proxy of the level of specialization)
t_{rq}^i	-	Interregional trade coefficients
r_i	-	adjustment factor of rows (*i*) of the additive RAS matrix balancing procedure
r_j	-	adjustment factor of columns (*j*) of the additive RAS balancing procedure

The SCGE model

Quantities (in order of appearance)

$L_{r,j}$	-	Labor demand in region *r* by industry *j*
$K_{r,j}$	-	Capital demand in region *r* by industry *j*
$VA_{r,j}$	-	Value added in region *r* in industry *j*
$XR_{r,j}$	-	Regional composite output of industry j
$XIM_{r,j}$	-	Industrial (international) import by industry j in region r
$X_{r,j}$	-	Domestic output by industry *j* in region *r*
$XT_{r,j}$	-	Total domestic output by industry *j* in region *r* (with inventories)
$EX_{r,j}$	-	Regional (international) export by industry *j*
$D_{r,j}$	-	Domestic supply of industry *j* in region *r* (without inventories)
$QR_{r,q,j}$	-	Interregional demand for industry j (point of origin: r, destination: q)
$Q_{r,j}$	-	Regional demand for industry *j* in region *r*
$LI_{r,j}$	-	Labor supply in region *r* to industry *j*
$KI_{r,j}$	-	Capital supply in region *r* to industry *j*
KR_r	-	Capital supply in region *r*
KN	-	Total national capital supply
$CR_{r,i}$	-	Regional industrial consumption by households
C_r	-	Regional composite household consumption demand
CIM_r	-	Regional import by households
$CTOT_r$	-	Total regional composite consumption
$IR_{r,i}$	-	Regional sectoral investment demand
I_r	-	Regional composite investment demand
IIM_r	-	Regional imported investment
$ITOT_r$	-	Total regional composite investment
$GR_{r,i}$	-	Regional sectoral government demand
GIM_r	-	Regional imported government purchases
$GTOT_r$	-	Total regional composite government demand
$EXNAT_i$	-	National export by industries

Prices (in order of appearance)

$PL_{r,j}$	-	Regional price of labor input
$PK_{r,j}$	-	Price of capital input
$PTL_{r,j}$	-	Labor price including taxes
$PTK_{r,j}$	-	Capital price including taxes
$PVA_{r,j}$	-	Price index of value added
$PTVA_{r,j}$	-	Price index of value added (including production taxes)
$PXR_{r,j}$	-	Price index of regional output
PIM_r	-	Price index of international import
$PX_{r,j}$	-	Price index of domestic regional output
$PEX_{r,j}$	-	Price index of regional industrial export
$PD_{r,j}$	-	Price index of domestic regional supply
$PQ_{r,j}$	-	Price index of regional final demand by industries
$PCR_{r,j}$	-	Price index of consumption (including consumption-specific commodity tax)
PC_r	-	Price index of domestic total consumption
$PCTOT_r$	-	Price index of total regional consumption
$PIR_{r,j}$	-	Price index of investment (including investment-specific commodity tax)
PI_r	-	Price index of domestic regional investment
$PITOT_r$	-	Price index of total regional investment
$PGR_{r,j}$	-	Price index of government demand (including commodity tax)
$PGTOT_r$	-	Price index of total regional government purchase
$PTEX_{r,j}$	-	Price index of regional export (including commodity tax)
$PEXNAT_j$	-	Price index of national export (in domestic currency)
PLR_r	-	Price of regional labor supply
PKR_r	-	Price of regional capital supply
PKN	-	Price of national capital supply (numeraire)

Nominal variables (in order of appearance)

YH_r	-	Household income
BH_r	-	Consumption budget
SH_r	-	Household savings
YG	-	Government revenues
$GNAT$	-	Government expenditures
SG	-	Government deficit
$TAXFAC$	-	Tax revenues on factors of production
$TAXPROD$	-	Tax revenues from production taxes
$TAXCOM$	-	Tax revenues from commodity taxes
ER	-	Exchange rate
$INVNAT$	-	Total investment expenditure (total national savings)

Exogenous parameters for the closure of the model (in alphabetical order)

PWM_r	-	World price of import
PWE_j	-	World price of export
$CIV_{r,j}$	-	Changes in domestic inventories in industry *j* in region *r* (with negative sign)
$CIVM_r$	-	Changes in imported inventories in region *r*

LS_r - Regional labor supply (updated by interregional migration)
KS_r - Regional capital supply (updated by capital accumulation)
SW - Current account

Model parameters

$a_{r,j}^{CD}$ - shift parameter in value added (considered as TFP in the SCGE model)
$\alpha_{r,j}$ - share parameter of labor in Cobb-Douglas production function
$\beta_{r,j}$ - share parameter of capital in Cobb-Douglas production function
$a_{r,i,j}$ - input coefficient of input *i* in industry *j*
$a_{r,j}^{VA}$ - input coefficient of value added in Leontief production function

$d_{r,j}^{X1}$ - shift parameter in total regional output CES
$b_{r,j}^{XR}$ - share parameter of local output in regional output CES
$b_{r,j}^{XIM}$ - share parameter of industrial import in regional output CES
$\sigma_{r,j}^{X1}$ - elasticity of substitution in total regional output
$\rho_{r,j}^{X1}$ - elasticity parameter

$d_{r,j}^{X2}$ - shift parameter of CET function
$b_{r,j}^{EX}$ - share parameter of export in the CET function
$b_{r,j}^{D}$ - share parameter of domestic supply in the CET function
$\sigma_{r,j}^{X2}$ - elasticity of transportation
$\rho_{r,j}^{X2}$ - elasticity parameter

$d_{r,i}^{Q}$ - shift parameter of the interregional CES demand function
$b_{q,r,i}^{QR}$ - share parameter of interregional demand
$\sigma_{r,i}^{Q}$ - elasticity of substitution in the interregional CES demand function
$\rho_{r,i}^{Q}$ - elasticity parameter

d_{r}^{LR} - shift parameter in CET
$b_{r,j}^{LI}$ - share parameter of regional industry level labor supply in CET
σ_{r}^{LR} - elasticity of transformation in CET
$\rho_{r,j}^{LR}$ - elasticity parameter

d_{r}^{KR} - shift parameter in CET
$b_{r,j}^{KI}$ - share parameter of regional industry level capital supply in CET
σ_{r}^{KR} - elasticity of transformation in CET
$\rho_{r,j}^{KR}$ - elasticity parameter

d^{KN} - shift parameter in CET
b_{r}^{KR} - share parameter of regional capital supply in CET
σ^{KN} - elasticity of transformation in CET
ρ^{KN} - elasticity parameter

d_{r}^{C} - shift parameter in CES regional domestic consumption demand
$b_{r,i}^{CR}$ - share parameter of sectoral consumption demand in CES
σ_{r}^{C} - elasticity of substitution of sectoral consumption demand
ρ_{r}^{C} - elasticity parameter

d_{r}^{CTOT} - shift parameter in total regional consumption demand CES function

b_r^C	-	share parameter of domestic composite consumption
b_r^{CIM}	-	share parameter of import consumption
σ_r^{CTOT}	-	elasticity of substitution in composite CES consumption demand function
ρ_r^{CTOT}	-	elasticity parameter
d_r^I	-	shift parameter in total regional investment demand CES function
$b_{r,i}^{IR}$	-	share parameter of sectoral investment demand in CES
σ_r^I	-	elasticity of substitution of sectoral investment demand
ρ_r^I	-	elasticity parameter
d_r^{ITOT}	-	shift parameter in total regional investment demand CES function
b_r^I	-	share parameter of domestic composite investment
b_r^{IIM}	-	share parameter of import investment
σ_r^{ITOT}	-	elasticity of substitution in composite CES investment demand function
ρ_r^{ITOT}	-	elasticity parameter
$a_{r,i}^G$	-	share parameter of domestic government demand
a_r^{GIM}	-	share parameter of import government purchases
d_j^{EXNAT}	-	shift parameter of national export demand CES
$b_{r,j}^{EXR}$	-	share parameter of national sectoral export
σ_j^{EXNAT}	-	elasticity of substitution of national sectoral export
ρ_j^{EXNAT}	-	elasticity parameter
$\tau_{q,r,i}$	-	Iceberg type transportation cost
$tlab_{r,j}$	-	Labor tax rate
$tcap_{r,j}$	-	Capital tax rate
$tprod_{r,j}$	-	Production tax rate
$tcomCR_{r,i}$	-	Commodity tax rate in consumption
$tcomIR_{r,i}$	-	Commodity tax rate investment
$tcomGR_{r,i}$	-	Commodity tax rate government purchases
$tcomXIR_{r,i}$	-	Commodity tax rate in intermediate use
$tcomEX_{r,i}$	-	Commodity tax rate in export
$tcomCIV_{r,i}$	-	Commodity tax subsidies on changes in inventories
sL_r	-	marginal propensity of saving of labor income
sK_r	-	marginal propensity of saving of capital income
si_r	-	parameter for distribution of national savings between regions
sg_r	-	parameter for distribution of government expenditures between regions

Dynamic parameters of capital accumulation and migration

δ	-	depreciation rate of capital stock
$LMIGR_{r,t}$	-	net labor migration in region *r* in time period *t*
$NMIGR_{r,t}$	-	net population migration in region *r* in time period *t*
$U_{r,t}$	-	Utility in region *r* in time period *t*
$H_{r,t}$	-	Housing
$N_{r,t}$	-	Population
φ	-	sensitivity of labor migration to utility differences

θ - sensitivity of labor migration
$initC_r$ - initial consumption in the utility function determining labor migration
αH - share parameter of housing per capita in the utility function
βH - share parameter of real consumption per capita in the utility function

The TFP model

$TFP_{r,t}$ - total factor productivity

The macro model

b_t - government debt per GDP
$\overline{b}$ - long-term desired rate of government debt per GDP
ed_t - government deficit per GDP
$\overline{ed}$ - long-term desired rate of government deficit per GDP
i - interest rate (exogenous)
π - inflation rate (calculated in the SCGE model)
g - GDP growth rate (calculated in the SCGE model)
adj - adjustment parameter for consistency between calculated and actual deficit

A.2. List of activities

- Agriculture (A)
- Mining and quarrying (B)
- Manufacture of food products, beverages and tobacco products (C 10, 11, 12)
- Manufacture of textiles, wearing apparel and leather products (C 13, 14, 15)
- Manufacture of wood and of products of wood, except furniture, paper and paper product and printing and reproduction of recorded media (C 16, 17, 18)
- Manufacture of coke and refined petroleum products (C 19)
- Manufacture of chemicals and chemical products (C 20)
- Manufacture of basic pharmaceutical products and pharmaceutical preparations (C 21)
- Manufacture of rubber and plastic products and other non-metallic mineral products (C 22, 23)
- Manufacture of basic metals and fabricated metal products (C 24, 25)
- Manufacture of computer, electronic and optical products (C 26)
- Manufacture of electrical equipment (C 27)
- Manufacture of machinery and equipment n.e.c. (C 28)
- Manufacture of motorvehicles and other transport equipments (C 29, 30)
- Other manufacturing, repair and installation of machinery and equipment (C 31, 32, 33)
- Electricity, gas, steam and air conditioning supply (D)
- Water collection, treatment and supply (E 36)
- Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services (E 37, 38, 39)
- Construction (F)
- Wholesale and retail trade; repair of motor vehicles and motorcycles (G)
- Transportation and storage (H)
- Accommodation and food service activities (I)
- Publishing activities, motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities (J 58, 59, 60)

- Telecommunications (J 61)
- Computer programming, consultancy and related activities; information service activities (J 62, 63)
- Financial and insurance activities (K)
- Real estate activities (L)
- Legal and accounting activities; activities of head offices; management consultancy activities and architectural and engineering activities; technical testing and analysis (M 69, 70, 71)
- Scientific research and development (M 72)
- Advertising and market research and other professional, scientific activities (M 73, 74, 75)
- Administrative and support service activities (N)
- Public administration and defense; compulsory social security (O)
- Education (P)
- Human health activities (Q 86)
- Social work activities (Q 87,88)
- Arts, entertainment and recreation (R)
- Other services activities (S)

** "Activities of households as employers; undifferentiated goods- and service producing activities of households for own use" as a sector is not accounted for in the analysis for the sake of simplicity.*

A.3. List of NUTS III regions

- Borsod-Abaúj-Zemplén (BA)
- Békés (BE)
- Bács-Kiskun (BK)
- Budapest (BP)
- Baranya (BR)
- Csongrád (CS)
- Fejér (FE)
- Győr-Moson-Sopron (GY)
- Hajdú-Bihar (HB)
- Heves (HE)
- Jász-NagyKun-Szolnok (JA)
- Komárom-Esztergom (KE)
- Nógrád (NO)
- Pest (PE)
- Somogy (SO)
- Szabolcs-Szatmár-Bereg (SZ)
- Tolna (TL)
- Vas (VA)
- Veszprém (VE)
- Zala (ZA)

A.4. Generation of the interregional input-output matrix

In this section we introduce a simple top-down non-survey method of how we can estimate an interregional input-output table using secondary data and the national input-output table. Our goal is to calculate a table which contains 20 'regions' (19 counties and the capital), their intraregional I-O linkages and their interregional relationships as well. The goal of the estimation is to create the database that can be used for the calibration of the SCGE block of GMR Hungary model (introduced in Section 4).

The base of our estimation is the Hungarian supply and use tables, value added, consumption, investment and employment by industry at the national level. Initially these tables contain 65 NACE rev. 2 industries and 65 CPA product classes. We keep the number of product classes but we need to aggregate the number of activities to 37 (due to the available regional data). We also use data at the county level, these are the following: value added, consumption, investment and number of employees by economic activities. Besides, we apply in the model the population data by districts and the consumption expenditures per capita (COICOP) at the NUTS 2 level. All these data are from 2010 and available in the dissemination database of the Hungarian Central Statistical Office (KSH).[15]

List of data:

- Use table for domestic output at basic prices NACE Rev. 2 (ESA2010)
- Supply table at basic prices, including the transformation into purchasers' prices NACE Rev. 2 (ESA2010)
- Per capita final consumption expenditure by purpose (COICOP) (NUTS II level, per capita, HUF, aggregated)
- Population (NUTS III level)
- Regional gross fixed capital formation, NUTS II. (million HUF, 11 aggregated NACE industry groups)
- Gross value added (million HUF, aggregated, NUTS III level)
- Number of employees by activities (capita, NUTS III level, 37 NACE 2 industries)

In the next five sub-chapters we introduce the different sequential steps of regionalization method. First, we estimate regional supply and use tables with net interregional trade volumes. Then we account for gross trade volumes using the CHARM method. In the second step we transform these supply and use matrices into symmetric input-output tables using absorption and market share matrices. Third, we estimate the interregional origin and destination of trade by employing the gravity model of trade. Fourth, following the Chenery-Moses model we determine the user industry of interregional trade. Fifth, we eliminate remaining inconsistencies from the interregional input-output table by balancing it using RAS technique.

A.4.1. One-region regionalization

In the first step, we estimate the regional supply and use tables, which will serve as the basis of the estimation of an interregional symmetric input-output table. First, we use the method introduced by Jackson (1998) which does not need a wide variety of detailed regional data, thus it can be applied to Hungarian NUTS 3 regions. The key elements of this method is the regionalization factors which will be used to derive the regional supply and use tables from their national counterparts. Since the regionalization relies on available regional statistical data which is only available in industry structure we apply the following simple industrial ratio:

$$\varepsilon_i^R = \frac{Emp_i^R}{Emp_i^N} \tag{113}$$

[15] http://statinfo.ksh.hu/Statinfo/themeSelector.jsp?&lang=en

where Emp_i^R and Emp_i^N stand for the number of employees in the regional (*R*) and national (*N*) industries (*i*). Thus the regionalization factor (ε_i^R) expresses the share of regional industrial employees. The reason for the use of the number of employees is that regional data in Hungary is generally not detailed enough thus this is the only set of data which has a great industrial resolution even at the NUTS 3 level. Unfortunately, there is a trade-off between industrial depth and comprehensiveness. While employment would be a better source of data since it accounts for a broader concept of employees it is available only for a small number of aggregated industries at the regional level. Thus we chose the number of employees as the basis of the estimation which accounts only for around ¾ of the total employment but its industrial resolution is much higher. As a consequence of poor data availability, we need to approximate regional gross output as well for which we use the regionalization factor from equation (113):

$$x_i^R = \varepsilon_i^R x_i^N \tag{114}$$

where *i* denotes industries, x_i^N and x_i^R shows the national and regional industrial gross output value.

We also use this ratio to the generation of the regional supply and use tables, although it can cause bias in the regions/industries with different labor and capital intensity. Nevertheless, there is no other data available, which is detailed enough to be applied in the required depth. The regionalization of the supply table is described by the following equation:

$$V_{i,k}^R = \varepsilon_i^R \cdot V_{i,k}^N \tag{115}$$

Where $V_{i,k}^N$ is national supply table. The regionalization of the use table starts with the commodity use of industries:

$$U_{k,i}^R = U_{k,i}^N \cdot \varepsilon_i^R \tag{116}$$

where *k* denotes commodities and $U_{k,i}^N$ is the transaction block of the national use table. Although the official tables are in *commodity X industry* structure we transposed the supply table in order to be consistent with Jackson (1998) and because it is a prerequisite of Chapter A.4.2 (see later). Thus in equation (115) the supply table is multiplied by the regionalization factor from the left hand side while the use table is multiplied from the right hand side. As a result, we regionalize these tables along the industry dimension assuming that the commodity structure of the output of regional industries is identical to the national ones. Similar argument can be given in case of industrial use, meaning that national and regional technologies are identical.

Next, we regionalize the remaining blocks of the use table. First, with the use of the regionalization factor we regionalize the block of the value added (including compensation of employees, other net taxes on production and operating surplus).

$$VA_{v,i}^R = VA_{v,i}^N \cdot \varepsilon_i^R \tag{117}$$

Where v denotes the three above mentioned components of the value added block. Finally, the taxes less subsidies on products and the foreign import by industries rows are needed to be regionalized:

$$TAX_i^R = TAX_i^N \cdot \varepsilon_i^R \tag{118}$$

$$IMP_i^R = IMP_i^N \cdot \varepsilon_i^R \tag{119}$$

At this point of the regionalization the use by industries is complete, now we turn our attention to final use. Because of the lack of data, we estimate the values of the final use block by preserve the national commodity structure (obtained from the national use table):

$$FD_{k,m}^R = \frac{FD_{k,m}^N}{FD\_TOT_m^N} \cdot FD\_TOT_m^R \tag{120}$$

$$FD_{k+1,m}^R = \frac{FD_{k+1,m}^N}{FD\_TOT_m^N} \cdot FD\_TOT_m^R \tag{121}$$

$$FD_{k+2,m}^R = \frac{FD_{k+2,m}^N}{FD\_TOT_m^N} \cdot FD\_TOT_m^R \tag{122}$$

where m denotes the type of use (e.g. households, investment, government, etc.), $FD_{k,m}^N$ denotes national final use by commodities (k) and final users (m) based on the use table and $FD\_TOT_m^N$ and $FD\_TOT_m^R$ is the total use by national and regional final users (without commodity dimension). In equation (121) and (122) we introduce to new rows into $FD_{k,m}^R$ matrices, where *k+1* and *k+2* rows contain the expenditure of regional final users on foreign imports and tax less subsidies on products.

The total final use is determined differently in each cases. In case of households which is calculated using NUTS 2 per capita consumption expenditure and population data. In this case we assumed that each NUTS 3 region within a NUTS 2 region has the same per capita consumption level. In case of gross capital formation we used regional gross fixed capital formation data from the National Statistical Office. The total regional value of changes in inventories, total use by government is given by regionalizing the national value by regional total GDP shares. Foreign export values were regionalized based on regional total output shares.

At this point, the equality of the industrial supply and use between the two tables still holds, because we corrected them with the same regionalization factors. However, this is not warranted in case of the commodity dimension. According to Jackson (1998) we account for this 'inconsistency' as net interregional trade. We calculate the total regional use by commodities, which consists of two factors: intermediate use and final use (including export). Then we extract the total supply by commodities (based on the supply table) from the total use. Where total supply consists only of domestic supply since both tables depict domestic supply and use. The formulation of the equation can be expressed as follows:

$$IM_k^R = \left(\sum_i U_{ki}^R + \sum_m FD_{km}^R\right) - \sum_i V_{ik}^R \tag{123}$$

where IM_k^R stands for the interregional net import of commodity k. If the value generated by this formula is positive, then regional use of a given commodity is greater than its supply, thus the region needs import from the other parts of the country. Hence we account for this import in the row of interregional import of the regional supply table. In order to avoid negative values in the table and complications later with model calibrations. Otherwise, if the generated value is negative, the region has surplus of supply of the given commodity. We take this surplus into account as interregional export in the block of final use of the use table. The tables are not only connected to the foreign countries but to other regions of the country as well, by interregional trade. However, at the end of this step the row and column of the interregional trade in the tables show only the amount of the trade by commodities between the region and rest of country, without giving any information about the origin and destination industry and region of export. On the other hand, applying this method to 20 Hungarian NUTS 3 regions we get interregionally consistent trade data where the value of total interregional import is equal to the value of total interregional export.

Jackson (1998) method only provides net trade volumes which is a small portion of actual gross trade volumes. The determination of these volumes would be crucial in case of a regional CGE model in order to depict the size of interdependency of regions on interregional trade. Thus we employ the method introduced by Kroenenberg (2007), the Cross-Hauling Adjusted Regionalization Method (CHARM), which is a method using international trade data to estimate product heterogeneity and re-export in order to estimate gross trade volumes. Kroenenberg's initial purpose was to create a method which is suitable to estimate the level of cross-hauling using national product heterogeneity measures (Flegg et al., 2014). First, it is assumed that cross-hauling (simultaneous import and export in case of the same commodity) is a function of product heterogeneity because interregional trade is motivated by heterogeneous products. In a fictional world where all products are homogenous, there would be no intention for simultaneous export and import. On the other hand Korenenberg also supposes that re-export is also affected by the size of region. The formulation of cross-hauling at the national level (in case of foreign trade) can be expressed as follows:

$$CH_k^N = h_k^N \cdot (X_k^N + Z_k^N + D_k^N) \tag{124}$$

where cross-hauling (CH_k^N) by commodities is a function of product heterogeneity (h_k^n), total output (X_k^N), total intermediate use (Z_k^N) and final demand (D_k^N). Product heterogeneity can be expressed using the equation on the right-hand side:

$$h_k^N = \frac{EXP_k^N + IMP_k^N - |EXP_k^N - IMP_k^N|}{2 \cdot (X_k^N + Z_k^N + D_k^N)} \tag{125}$$

where we subtract the absolute value of net trade from the gross trade volume ($EXP_k^N + IMP_k^N$) in order to calculate the size of crosshauling in case of foreign trade which is actually twice the size of cross-hauling that is why we divide it by two in the denominator (among the rest of variables). Once the national product heterogeneity is calculated we can use it to calculate the regional size of cross-hauling by employing it to regional data:

$$CH_k^R = h_k^N (X_k^R + Z_k^R + D_k^R) \tag{126}$$

Finally, we are able to calculate the estimated gross volumes of interregional trade (imports and exports) in each region:

Ha $IM_k^R > 0$

$$IMP\_GR_k^R = IM_k^R + CH_k^R$$

(127)

$$EXP\_GR_k^R = CH_k^R$$

(128)

Ha $IM_k^R < 0$

$$IMP\_GR_k^R = CH_k^R$$

(129)

$$EXP\_GR_k^R = \left|IM_k^R\right| + CH_k^R$$

(130)

At this point we created regional supply and use tables for each region with gross regional trade volumes. But with the introduction of cross-hauling our tables remain consistent internally, meaning that the total use equals the total supply but their national aggregation will be more than the actual data since we did not subtract cross-hauling from the internal cells of the use table.

A.4.2. Generating the symmetric tables

In order to transform the supply and use tables into symmetric input-output tables, we follow Miller – Blair (2009)[16] and we calculate two new matrices: the market share matrix (D) and the absorption matrix (B). The market share matrix is a coefficient matrix derived from the supply table, and shows the share of the industries in the production of a commodity. Hence the columns of the (transposed) supply table are divided by the sum of the columns, namely by the supply of the commodities. It can be expressed as:

$$D^R = V^R \widehat{q^R}^{-1}$$

(131)

where q^R is the output by commodities. The absorption matrix is derived from the use table, and shows the share of the commodities in the total use of an industry. Hence the columns of the use table are divided by the industrial outputs. It is shown by the following equation.

$$B^R = U^R \widehat{X^R}^{-1}$$

(132)

Where $\widehat{X^R}$ is a diagonal matrix of regional industrial gross output. By the help of these two calculated matrices, we can calculate the coefficient form of the symmetric input-output table as follows:

$$A^R = D^R \cdot B^R$$

[16] For more details see Chapter 5

(133)

As a result we fix the shares in the market share and absorption matrices. Thus we assume that industry *i* uses a fixed proportion of the products produced by industry *j*. In this new *industry X industry* structure matrix the first cell A^R (1,1) shows how many units of agricultural output (commodities produced altogether by the agricultural sector) is needed as input to produce one unit of agricultural output. Correspondingly, the cell A^R (1,2) shows how many units of commodities produced by the agricultural industry is needed to produce one unit of the manufacturing industry's output. On the whole, A^R is an industry by industry structure table, which shows the amount of industrial inputs required by the industries of the local economy in order to produce one unit of output. This approach is called the industry technology (Miller and Blair, 2009).

The same procedure is undertaken in case of final demand using the market share matrix and the commodity coefficient matrix of final demand (FD^R):

$$A\_FD^R = D^R \cdot FD^R$$

(134)

As a results we get the coefficient matrix of industrial final demand.

Finally, we can calculate the industrial inter-industry transactions and final use matrices by multiplying A^R by industrial output and A_FD^R by total final use by final users.

$$Z_{i,j}^R = A_{i,j}^R \cdot x_j^R$$

(135)

$$Z\_FD_{i,m}^R = A\_FD_{i,m}^R \cdot FD\_TOT_m^R$$

(136)

A.4.3. Obtaining the origin and destination regions of trade – the gravity model

In this step, we intend to obtain the destination region of interregional export and the origin region of interregional import. We apply the gravity model (Black, 1972), which treats trade mechanism as Newton's law of universal gravitation. This means, that trade volumes are higher between regions that are geographically close to each other, and between regions that are larger in (economic) size (ceteris paribus). Since interregional import and export is already a part of the regionalized supply and use tables, they were transformed into industrial structure in Section A.4.2. Thus in this section we employ the gravity model determining interregional trade flows by industrial exports and imports. Interregional trade volumes are determined by the following equation:

$$T_{rs}^i = \frac{S_r^i K_s^i F_{rs}^i}{\sum_s K_s^i F_{rs}^i},$$

(137)

where T_{rs}^i is shipment of output of industry *i* between region *r* and region *s*, S_r^i is total shipment of the output of industry *i* leaving region *r* to other domestic regions. K_s^i is the total demand for the output of industry *i* in region *s* and finally, F_{rs}^i is the frictional factor, which is calculated as $1/d_{rs}^{\lambda}$, where d_{rs} is the geographical distance (via roads) between regions, and λ^i is an industry-specific factor showing the sensitivity of trade for the distance. Speaking in terms of the gravity model S_r^i is the 'size' of one region, while K_s^i is the 'size' of the other region and d_{rs} is the distance between them (in the form of the friction parameter).

These factors are derived from the first two steps, except the frictional factor. The total shipment equals the industry structure interregional export of region *r* calculated in Section A.4.2, while the total (intermediate and final) demand of region *s* is given again by the aggregation of the regional I-O table(s) derived in Section A.4.2. However, in order to determine the friction parameter, first we need to estimate the distance sensitivity (λ^i). Black (1972) applied the following regression for the estimation of λ^i:

$$\ln(\lambda_i + 1) = 0.05701 + 1.038 \cdot LM_i - 0.511 \cdot CP_i \,, \tag{138}$$

where LM_i is the ratio of the national intraregional demand for output *i* and the national local output:

$$LM_i = \frac{\sum_r S_i^{rr}}{\sum_{r,s} S_i^{rs} + \sum_r S_i^{rr}} \tag{139}$$

CP^i serves as the approximation of regional specialization. By assumption larger specialization means a higher chance of interregional trade. The index shows the share of the largest exporting region of exporting interregionally output *i*:

$$CP_i = \frac{max(\sum_s S_i^{rs})}{\sum_{r,s} S_i^{rs}} \tag{140}$$

We must note that the original equation was estimated in 1972 in the USA, thus its values could be taken into account with caution in the circumstances in Hungary in 2010. Unfortunately, (to our knowledge) not such estimation was carried out in Hungary.

If we consider the right-hand side of equation (137) without S_r^i , we can notice that this ratio is the regional demand to total demand ratio corrected by the frictional factor. Namely, we calculate the distribution of frictional demand between the destination regions. By multiplying these coefficients by S_r^i we generate the size of the transportation of good i between the regions. Thus, we allocate all of the export of product i of region r to the other regions.

The gravity model is capable of distributing interregional export (shipment) to different regions based on distance, friction and economic size. However, there is no guarantee that this interregional structure of trade will be consistent with the total interregional import of different regions, calculated in Section A.4.1. Thus in order to maintain consistency further balancing of the trade matrix will be required. Before that last step however, we further extend the dimensions of interregional trade by introducing the origin and destination industry of interregional trade in the next section.

A.4.4. Obtaining the origin and destination sector of the interregional trade

In this step, we further extend our interregional trade matrix in the direction of source of demand (industries, final users). We use assumptions of the Chenery-Moses model (Moses, 1955), which allows for the sectors to export not only to other sectors but also directly to final users. From the trade data generated in the previous step, we calculate trade coefficients based on the Chenery-Moses model. First we calculate industrial trade coefficients:

$$t_{rs}^{i} = \frac{T_{rs}^{i}}{\sum_r T_{rs}^{i}},$$

(141)

where T_{rs}^{i} is one arbitrary element from the trade matrix calculated in Section A.4.3. The import coefficient t_{rs}^{i} shows the share of imports supplied by region *r* in region *s* in case of the products of industry *i*. Table 7 is an illustration of the coefficient matrix. It is worth to notice, that the sum of the regions inside a sector equals to one (in the case of the second region: $t_{12}^{i} + t_{22}^{i} + t_{32}^{i} = 1$).

Table 7: Illustration of the coefficient matrix in case of three industries

User region Supply region		1.region	2.region	3.region
Agriculture	1.region	t_{11}^{Agr}	t_{12}^{Agr}	t_{13}^{Agr}
	2.region	t_{21}^{Agr}	t_{22}^{Agr}	t_{23}^{Agr}
	3.region	t_{31}^{Agr}	t_{32}^{AGr}	t_{33}^{Agr}
Manufacturing	1.region	t_{11}^{Man}	t_{12}^{Man}	t_{13}^{Man}
	2.region	t_{21}^{Man}	t_{22}^{Man}	t_{23}^{Man}
	3.region	t_{31}^{Man}	t_{32}^{Man}	t_{33}^{Man}
Services	1.region	t_{11}^{Ser}	t_{12}^{Ser}	t_{13}^{Ser}
	2.region	t_{21}^{Ser}	t_{22}^{Ser}	t_{23}^{Ser}
	3.region	t_{31}^{Ser}	t_{32}^{Ser}	t_{33}^{Ser}

Thereafter, we form diagonal matrices from the interregional trade coefficients (between each region *r* and *q* pair):

$$\hat{T}_{r,q} = \begin{bmatrix} t_{r,q}^{1} & 0 & 0 \\ 0 & t_{r,q}^{2} & 0 \\ 0 & 0 & t_{r,q}^{i} \end{bmatrix}$$

(142)

Where *r* stands for the origin region of trade and *q* stands for the destination region of trade, while *i* stands for the industrial origin of trade. Using these diagonal matrices we created a matrix of interregional trade coefficients in each region pair as follows:

$$TT = \begin{bmatrix} 0 & \hat{T}_{12} & \dots & \hat{T}_{1r} \\ \hat{T}_{21} & 0 & \dots & \hat{T}_{2r} \\ \dots & \dots & & \dots \\ \hat{T}_{r1} & \hat{T}_{r2} & \dots & 0 \end{bmatrix}$$

(143)

where each element of the trade matrix is an *industry by industry* trade matrix describing the share of output *i* transported from one region to another. We must note that intraregional trade volumes are not accounted for in this matrix since we already estimated those values in Section A.4.1 and A.4.2.

Now we intend to approximate the interregional inter-industry trade of the interregional input-output table.

First, we employ the following equation where the interregional trade matrix is multiplied by the national input-output coefficients and the regional gross output values (approximated in Section A.4.1).

$$IIO = \begin{bmatrix} 0 & \hat{T}_{12} & \dots & \hat{T}_{1r} \\ \hat{T}_{21} & 0 & \dots & \hat{T}_{2r} \\ \dots & \dots & & \dots \\ \hat{T}_{r1} & \hat{T}_{r2} & \dots & 0 \end{bmatrix} \cdot \begin{bmatrix} A_N & & & \\ & A_N & & \\ & & \dots & \\ & & & A_N \end{bmatrix} \cdot \begin{bmatrix} \widehat{X_1} & & & \\ & \widehat{X_2} & & \\ & & \dots & \\ & & & \widehat{X_r} \end{bmatrix} \tag{144}$$

$$IIO = \begin{bmatrix} 0 & \hat{T}_{12} A_N \widehat{X_2} & \dots & \hat{T}_{1r} A_N \widehat{X_r} \\ \hat{T}_{21} A_N \widehat{X_1} & 0 & \dots & \hat{T}_{2r} A_N \widehat{X_r} \\ \dots & \dots & & \dots \\ \hat{T}_{r1} A_N \widehat{X_1} & \hat{T}_{r2} A_N \widehat{X_2} & \dots & 0 \end{bmatrix} \tag{145}$$

Where *IIO* matrix stands for the total interregional inter-industry trade volumes (*region x industry x region x industry*). For the first sight it could be surprising that we use national coefficients since we already estimated regional ones in Section A.4.2. While it is true, we did it only to estimate interregional trade volumes that served as inputs of the gravity model, thus we initially intended to use only the estimated trade data to approximate the final structure of interregional inter-industry trade volumes in this section.

The same procedure can be undertaken in case of the trade for final use purposes:

$$IFD = \begin{bmatrix} 0 & \hat{T}_{12} & \dots & \hat{T}_{1r} \\ \hat{T}_{21} & 0 & \dots & \hat{T}_{2r} \\ \dots & \dots & & \dots \\ \hat{T}_{r1} & \hat{T}_{r2} & \dots & 0 \end{bmatrix} \cdot \begin{bmatrix} A\_FD_N & & & \\ & A\_FD_N & & \\ & & \dots & \\ & & & A\_FD_N \end{bmatrix} \cdot \begin{bmatrix} \widehat{FD\_TOT_1} & & & \\ & \widehat{FD\_TOT_2} & & \\ & & \dots & \\ & & & \widehat{FD\_TOT_r} \end{bmatrix} \tag{146}$$

$$IFD = \begin{bmatrix} 0 & \hat{T}_{12} A\_FD_N \widehat{FD\_TOT_2} & \dots & \hat{T}_{1r} A\_FD_N \widehat{FD\_TOT_r} \\ \hat{T}_{21} A\_FD_N \widehat{FD\_TOT_1} & 0 & \dots & \hat{T}_{2r} A\_FD_N \widehat{FD\_TOT_r} \\ \dots & \dots & & \dots \\ \hat{T}_{r1} A\_FD_N \widehat{FD\_TOT_1} & \hat{T}_{r2} A\_FD_N \widehat{FD\_TOT_2} & \dots & 0 \end{bmatrix} \tag{147}$$

Where *IFD* matrix (*region x industry x region x final users*) shows the total interregional trade for final demand purposes (households, government, gross capital formation, etc.). These matrices can be extended with the intraregional trade of inter-industry and final demand purpose trade volumes in order to complete the interregional transactions. Thus in the end, we get an interregional input-output table, which can represent the product flows between the region's sectors and final users. However, as mentioned before this table is not balanced, since the import side of interregional trade was not accounted for so far. In the next step we employ a common method to overcome this imbalance.

Our final imbalanced interregional table takes the form of the following (with the dimensions of *region x industry x region x (industry+final users)*):

$$Z^0 = [IIO, IFD]$$

(148)

$$Z^0 = \begin{bmatrix} 0 & \hat{T}_{12}A_N\widehat{X_2} & \dots & \hat{T}_{1r}A_N\widehat{X_r} & 0 & \hat{T}_{12}A\_FD_N\widehat{FD\_TOT_2} & \dots & \hat{T}_{1r}A\_FD_N\widehat{FD\_TOT_r} \\ \hat{T}_{21}A_N\widehat{X_1} & 0 & \dots & \hat{T}_{2r}A_N\widehat{X_r}, & \hat{T}_{21}A\_FD_N\widehat{FD\_TOT_1} & 0 & \dots & \hat{T}_{2r}A\_FD_N\widehat{FD\_TOT_r} \\ \dots & \dots & & \dots & \dots & \dots & & \dots \\ \hat{T}_{r1}A_N\widehat{X_1} & \hat{T}_{r2}A_N\widehat{X_2} & \dots & 0 & \hat{T}_{r1}A\_FD_N\widehat{FD\_TOT_1} & \hat{T}_{r2}A\_FD_N\widehat{FD\_TOT_2} & \dots & 0 \end{bmatrix}$$

(149)

Where all the intraregional intermediate and final use flows are calculated in Section 1.2, in the next step we only intend to eliminate the imbalances in the interregional flows.

A.4.5. Balancing the final interregional matrix

As mentioned in Section A.4.3, interregional trade estimation was based on an export-side approach which does not guarantee that the estimated interregional trade flow is consistent with the total interregional import values calculated in Section A.4.2. In this section we use a matrix balancing method to overcome this problem. Since our final table contains a number of negative cells (changes in inventories) the traditional RAS method would not work properly because when it increases the values in a row (by multiplying all values in the row with the same factor), it would actually decrease the the value of negative cells which is against its original logic. Furthermore, in extreme cases it can turn the total value of cells in a given row or column into the opposite sign by this adjustment. Thus we decided to employ a bi-proportional method that is capable of handling negative values as well: an appropriate method is called additive RAS which was developed by Révész (2001).

The logic of the method is the following. First we calculate the share of each cell in each row and column using their absolute values (since they can be negative). Then we distribute the difference between the actual sum of the table (according either to rows or columns, $\sum_j z_{i,j}$) and the desired frame (x_i and x_j).

Step 1 – rows:

$$r_i = \frac{|z_{i,j}^0|}{\sum_j |z_{i,j}^0|} \cdot \left(x_i - \sum_j z_{i,j}^0\right)$$

(150)

$$z_{i,j}^1 = z_{i,j}^0 + r_i$$

(151)

where the values in x_i are the regional industry output.

Step 2 – columns:

$$r_j = \frac{\left|z_{i,j}^1\right|}{\sum_i \left|z_{i,j}^1\right|} \cdot \left(x_j - \sum_i z_{i,j}^1\right)$$

(152)

$$z_{i,j}^2 = z_{i,j}^1 + r_j$$

(153)

where the values in x_j are the regional interregional imports by industries and final users.

By repeating step 1 and 2 iteratively we can quickly balance the matrix according to the given desired frame. This way we maintain the sign of the table and we make sure that all cells in a given row/column are decreased/increased even if they are negative. An additional advantage of the method is that it can generate an appropriate result even if the sum of a column/row is negative, where the traditional RAS fails to operate (because of divergence).

A.5. An illustrative example of the interregional SAM

Interregional SAM			Region 1: Activities (i1 i2 … i37)	Region 1: Production factors (LAB CAP)	Region 1: Institutions (HHD GOV INV CIV)	…	Region 20: Activities (i1 i2 … i37)	Region 20: Production factors (LAB CAP)	Region 20: Institutions (HHD GOV INV CIV)	ROW (Rest of the world)	Savings	Government (national)	Taxes on production	Taxes on commodities	TLAP	TCAP
Region 1	Activities	i1 i2 … i37	Intermediate goods		Final demand		Intermediate goods		Final demand (interregional import)	Export						
Region 1	Production factors	LAB CAP	Value Added (wage/profit)													
Region 1	Institutions	HHD		Factor income (wage/profit)												
Region 1	Institutions	GOV										Regional gov. expenditure				
Region 1	Institutions	INV									Regional inv. expenditure					
Region 1	Institutions	CIV	Changes in inventories				Changes in inventories			CIV export				Subsidies on CIV		
…																
Region 20	Activities	i1 i2 … i37	Intermediate goods		Final demand (interregional import)		Intermediate goods		Final demand	Export						
Region 20	Production factors	LAB CAP					Value Added (wage/profit)									
Region 20	Institutions	HHD						Factor income (wage/profit)								
Region 20	Institutions	GOV										Regional gov. expenditure				
Region 20	Institutions	INV									Regional inv. expenditure					
Region 20	Institutions	CIV	Changes in inventories				Changes in inventories			CIV export				Subsidies on CIV		
ROW (Rest of the world)			Regional import		Regional import		Regional import		Regional import	Re-export						
Savings					Private savings Gov. savings				Private savings Gov. savings	Foreign savings		Gov. Saving				
Government (national)													TPROD income	TCOM income		
Taxes on production			Taxes on production				Taxes on production									
Taxes on commodities			Taxes on commodities		Taxes on commodities		Taxes on commodities		Taxes on commodities	Taxes on export						
TLAP																
TCAP																