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Sustainable, intelligent and inclusive regional and city models

The city-region GMR-Hungary economic impact assessment modelling framework

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Technical report

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Introduction

The present paper describes the city-region version of the multisectoral GMR-Hungary model, which is an extended spatial computable general equilibrium (SCGE) model built for policy impact assessment purposes. The main structure of the model is borrowed from the county level version of the model. However in this case the local units are city-regions instead of counties, which is supposed to show the regional differences more precisely.

Since multisectoral spatial models require a huge amount of initial data, in the first chapters, we start with the description of the methods used for the generation of the interregional input-output data. *First we define the local units and the distances between them, and then we introduce the data applied in the model*. Following Jackson's (1998) and Kroenenberg's (2007) models we regionalize national supply and use tables and estimate detailed interregional trade data. Then we transform our results into symmetric interregional input-output table that contains all the regions. In order to get the origin and destination regions of the interregional trade, Black's (1972) gravity model is used and so that to get the origin and destination sector of the interregional trade the Chenery-Moses model (Moses, 1955) is applied. In the last step of regionalization we balance the interregional input-output matrix, which is one of the main dataset needed for our model calibration.

In the second chapter of this paper, we describe the structure and the main equations of the spatial general equilibrium model. It starts with an overview and then the production structure of the model, the role of interregional trade and transportation and the role of the commodity tax is shown. We explain the behavior of the main economic actors, the demand side of the model and the equilibrium conditions in an interregional framework. In the end, the model dynamics; the regional capital accumulation, the interregional domestic labour migration and the two other dynamic model blocks the TFP and the macro model block are also described. The TFP (total factor productivity) block is responsible for describing the regions productivity changes and capable of enumerating the possible impact of different human capital interventions. The macro block on the other side is responsible for updating government deficit according to a macro rule which prescribes that the government can increase spending at most at the rate of economic growth. The model is solved recursively for each time period thus all dynamic change serves as an 'intervention' at the beginning of the next period.

In the appendix at the end of the paper one can find detailed information on the variables, parameters of the model, the list of regions and industries, with additional information on the interregional SAM used for calibration and the defined regions in the model.

1. Generation of the interregional input-output matrix

1.1. Regionalization of the national input-output table

In this section we introduce a simple top-down non-survey method of how we can estimate an interregional input-output table using secondary data and the national input-output table. Our goal is to calculate a table which contains 41 'region' (city regions and joint counties), their intraregional I-O linkages and their interregional relationships as well.

The base of our estimation is the Hungarian supply and use tables, GDP, consumption, investment and employment by industry at the national level. Initially these tables contain 65 NACE rev. 2 industries and 65 CPA product classes. We kept the number of product classes but we needed to aggregate the number of activities to 39 (due to the available regional data). We also use data at the county level, these are the following: GDP, consumption, investment and employment by branch of economic activity. Besides, we apply in the model the population data by districts and the consumption expenditures per capita (COICOP) at the NUTS 2 level. All these data are from 2010 and available at the dissemination database of the Hungarian Central Statistical Office (KSH). In addition, for the regionalization of the national and county level data we apply employment data from QualiDat Kft. These data are collected from reports of the firms at the district level by branch of economic activity. We corrected them with data estimated from the database of KSH in eight branches, where the role of the public sector is high, like the science research and development sector, the public administration sector, the education sector, healthcare, etc. (NACE: M 72, N, O, P, Q 86, Q88, R, S).

From these data, we estimate the regional consumption, employment, GDP and investment, which fit to the model structure. We aggregate the population data from the district level to the regional level and multiply it by the consumption expenditures per capita (COICOP), thus we get the regional consumption. In order to estimate the regional employment by industry, we distribute the county level employment to the regions by location (each county contains one or two city-regions and one county-region). We calculate the regional level employment to the sum of the employees in the same county from the corrected data of QualiDat, and we use this ratio to distribute the employment data of KSH from the county level to the regional level. Accordingly, we can approximate the industrial structure of the city-regions and county-regions, and the estimated regional employment data preserves the consistency with the official employment data at the county level. In order to calculate the regional GDP and regional investment, we also distribute the county level data to the regional level. In this case, the applied ratio is the estimated regional employment to county level employment ratio.

List of data:

- Use table for domestic output at basic prices NACE Rev. 2 (ESA2010)
- Supply table at basic prices, including the transformation into purchasers' prices NACE Rev. 2 (ESA2010)
- Household actual final consumption expenditure by purpose (COICOP)
- Regional gross fixed capital formation, NUTS II. (million HUF)
- Gross value added (million HUF)
- Number of employees by activities (capita)

- Firm level data (Qualidat Kft.)

Step 1: One-region regionalization

In the first step, we estimate the regional supply and use tables, and transform them to symmetric input-output tables. We use the method of Jackson (1998) which does not need detailed regional data, so we can apply it for the Hungarian city-regions. The main element of this method is the regionalization factor, which is a simple ratio:

$$x_i^R = \varepsilon_i^R x_i^N, \tag{1}$$

where x shows the national (N) and regional (R) industry (i) output value, while ε is the regionalization factor, which is the regional to national industrial employment ratio:

$$\varepsilon_i^R = \frac{Emp_i^R}{Emp_i^N} \tag{2}$$

We also use this ratio to the generation of the regional supply and use tables, although it can cause bias in the regions/industries with different labor and capital intensity. Nevertheless, there is no other variable, which is detailed enough to apply. It is described by the following equations:

$$V^R = \widehat{\varepsilon_i^R} V^N \tag{3}$$

$$U^R = U^N \widehat{\varepsilon_i^R}, \tag{4}$$

where $\widehat{\varepsilon_i^R}$ is a diagonal matrix with the elements of ε_i^R in the main diagonal. Furthermore, V^R is national supply table and U^R is the national use table. Thus, we can generate the regional supply table by multiplying the national supply table from the left side (which means by sectors) by the diagonal matrix of the regionalization factor (equation (3)). Correspondingly, we can generate the regional use table by multiplying from the right side the national supply table by the diagonal matrix of the regionalization factor (equation (4)).

Next, we regionalize the remaining blocks of the table by applying the output-ratio (ε^{xR}).

$$\varepsilon^{xR} = \frac{x^R}{x^N} \tag{5}$$

$$M_k^R = \varepsilon^{xR} M_k^N, \tag{6}$$

where k is the index of commodities. First, we regionalize the foreign import from the supply table, and then we regionalize with the same method the block of the commodity taxes and the block of value added from the use table.

$$VA^R = VA^N \widehat{\varepsilon_i^R} \tag{7}$$

$$TAX^R = TAX^N \widehat{\varepsilon_i^R}\ , \tag{8}$$

For lack of data, we estimate the values of the final use block by preserve the national industry structure:

$$FD_{km}^{R} = \frac{FD_{ik}^{N}}{\sum_{k=1}^{n} FD_{km}^{N}} FD_{m}^{N} , \quad (9)$$

where m denotes the type of use (e.g. government). We generate the sum of final uses by the help of proxy-variables. In the case of households, we use the regional to national consumption ratio, for the government expenditures and the changes in inventories we employ the regional to national GDP ratio, for the investment we apply the regional to national investment ratio and at last for the export and the remaining cells we use the output ratio.

At this point, the equality of the output of the two tables still stands, because we corrected them with the same factors. In this regard the consistency still retains. This is not warranted in the case of commodities, therefore, we take this inconsistency as interregional trade. We calculate the total regional use by commodities, which consists of two factors: intermediate use and final use (with export). Then we extract the total supply from the total use. The total supply also consists of two parts: domestic supply and import. The formulation of the equation can be expressed as follows:

$$IM_{k}^{R} = \left(\sum_{i} U_{ki}^{R} + \sum_{n} FD_{kn}^{R}\right) - \sum_{i} V_{ik}^{R} - M_{k}^{R} , \quad (10)$$

If the value generated by this formula is negative, then the use of the region is greater than the supply of it, so the region needs import from the other parts of the country. Hence we take it into account in the row of interregional import of the regional supply table. Otherwise, if the generated value is positive, the region has surplus of supply of the given commodity. We take this surplus into account as interregional export in the block of final use of the use table. The tables we get with this method show which industry produces which commodities in a region, which amount of import from abroad comes to the region and which commodities are consumed by the different industries and final users in the region. The tables are not only connected to the foreign countries but to other regions of the country as well, by interregional trade. However, at the end of the first step the rows of the interregional trade of the tables show only the amount of the trade by commodities between the region and the other part of the country, without giving any information about the origin and the destination industry and region of the export.

We calculate the cross-hauling with Kroenenbergs (2007) Cross-Hauling Adjusted Regionalization Method (CHARM), which is a method using international trade data to estimate product heterogeneity and re-export. First, it is assumed that cross-hauling is a function of product heterogeneity because interregional trade is motivated by heterogenous products. In a fictional world where all products are homogenous, there would be no intention for simultaneous export and import. This method also supposes that re-export would also be affected by the size of region. The formulation of cross-hauling can be expressed as follows:

$$CH = \varepsilon(X + Z + D) , \quad (11)$$

where cross-hauling (CH) is a function of product heterogeneity (ε), total output (X), total intermediate use (Z) and total demand (D). ε can be expressed from the equation on the right-hand side:

$$\varepsilon = \frac{\mathrm{V} - |\mathrm{B}|}{2(\mathrm{X} + \mathrm{Z} + \mathrm{D})}, \tag{12}$$

where V is the total gross trade (import and export) and B is trade balance. The expression in the nominator stands for the difference between gross trade volume and the absolute value of the trade balance, which is in fact twice the cross-hauling (that is the reason for the multiplication by ½). ε should be estimated using national data, thus ε is only national and not region specific.

Step 2: Generating the symmetric tables

In order to transform the supply and use tables to symmetric input-output tables, we need to calculate two new matrices: Market share matrix (D) and absorption matrix (B). The market share matrix is a coefficient matrix derived from the supply table, and shows the share of the industries in the production of a commodity. Hence the columns of the supply table are divided by the sum of the columns, namely by the supply of the commodities. It can be expressed as:

$$D^R = V^R \widehat{q^R}^{-1}, \tag{13}$$

where q is the output by commodities. The absorption matrix is derived from the use table, and shows the share of the commodities in the total use of an industry. Hence the columns of the use table are divided by the industrial outputs. It is shown by the following equation.

$$B^R = U^R \widehat{x^R}^{-1}, \tag{14}$$

By the help of these two matrices, we can calculate the coefficient form of the symmetric input-output table.

$$A^R = D^R B^R \tag{15}$$

The first cell A^R (1,1) shows how many units of agricultural output (commodities produced by the agricultural sector) is needed as input to produce one unit of agricultural output. Correspondingly, the cell A^R (1,2) shows how many units of agricultural commodities is needed to produce one unit of the manufacturing industry's output. On the whole, A^R is a symmetric table, which shows the needed amount of inputs from the other sectors of the economy to produce one unit of output from a given industry. By doing so we drop the dimension of commodities.

Step 3: Obtaining the origin and destination regions of the interregional trade – the gravity model

In this step, our aim is to obtain the destination region of the interregional export and the origin region of the interregional import. We apply the gravity model (Black 1972), which treat the trade mechanism as Newton's law of universal gravitation. This means, that the regions, in which the demand/supply ratio is high, and are close to each other, will trade with each other with higher probability. The main equation of the gravity model is the following:

$$T_{rs}^{i} = \frac{S_r^i K_s^i F_{rs}^i}{\sum_s K_s^i F_{rs}^i}, \tag{16}$$

where T_{rs}^{i} is the sum of the commodities produced by the sector i in region r traded to region s, S_r^i is the sum of the products produces by the sector i, which are traded from region r to other regions of the country. K_s^i is the total demand for the products of sector k in region r. F_{rs}^i is the frictional factor, which is ${}^{1}/{}_{d_{rs}^{\lambda}}$, where d_{rs} is the distance between the regions, while λ^i is a factor showing the sensitivity of trade for the distance. λ^i is sector specific.

These factors are derived from the first two steps, except the frictional factor. For determining F_{rs}^k, we need to estimate λ^i. Black applied the following regression for estimating λ^i:

$$\ln(\lambda^i + 1) = 0.05701 + 1.038LM^i - 0.511CP^i, \tag{17}$$

where LM^i is the total demand for the products of sector i in the whole economy, and CP^i shows the concentration of the production of the sectoral commodity i, which serve as a proxy for the regional specialization. Thus, the larger is the specialization of a region, the greater is the probability of trade.

If we consider the right-hand side of the main equation of the gravity model without S_r^i, we can notice that this ratio is the regional demand to total demand ratio corrected by the frictional factor. Namely, we calculate the distribution of frictional demand between the destination regions. By multiplying these coefficients by S_r^i we generate the size of the transportation of good i between the regions. So, we allocate all of the export from the product i of region r to the other regions.

At this point, the data are consistent in respect of export, hence the sum of rows of the trade matrix is related perfectly to the previous export data. Although, it is not warranted in respect of import to the previous import data, thus later we need to balance the matrix.

Step 4: Obtaining the origin and destination sector of the interregional trade

In the next step, we obtain the origin and user industry of trade with the Chenery-Moses model (Moses, 1955), which allows for the sectors to export not only to other sectors but also directly to final users. From the trade data generated in the previous step, we calculate trade coefficients by the help of the Chenery-Moses model. This calculation is shown in the next equation.

$$t_{rs}^{i} = \frac{T_{rs}^{i}}{\sum_r T_{rs}^{i}}, \tag{18}$$

where T_{rs}^{i} is one optional element from the trade matrix calculated in step 3. The coefficient t_{rs}^{i} shows that which proportion of the demand for a given product i of a user region s satisfies the region r. The second table is an illustration of the coefficient matrix. It is worth to notice, that the sum of the regions inside a sector equals to one (in the case of the second region: $t_{12}^{i} + t_{22}^{i} + t_{32}^{i} = 1$).

Supply region	User region	1.region	2.region	3.region
Agriculture	1.region	t_{11}^{i}	t_{12}^{i}	t_{13}^{i}
	2.region	t_{21}^{i}	t_{22}^{i}	t_{23}^{i}
	3.region	t_{31}^{i}	t_{32}^{i}	t_{33}^{i}
Manufacturing	1.region	t_{11}^{i}	t_{12}^{i}	t_{13}^{i}
	2.region	t_{21}^{i}	t_{22}^{i}	t_{23}^{i}
	3.region	t_{31}^{i}	t_{32}^{i}	t_{33}^{i}
Services	1.region	t_{11}^{i}	t_{12}^{i}	t_{13}^{i}
	2.region	t_{21}^{i}	t_{22}^{i}	t_{23}^{i}
	3.region	t_{31}^{i}	t_{32}^{i}	t_{33}^{i}

Table 2: Illustration of the coefficient matrix

Thereafter, we form diagonal matrices from the coefficients, and structure them to the form of the first element of the right-hand side of the equation (19). In the matrix's main diagonal, there are the intraregional coefficients, which represent the ratio of the demand of the sectors in a region covered from domestic supply. The further non diagonal elements are the interregional coefficients, which represent the export/import relationships. The interregional trade data are derived using the regional output data and the technical coefficients matrices (A^R), which contains the region's intermediate uses. First, we multiply from the left side the diagonal matrix of the technical coefficients matrices by the trade coefficients. At this point, we separate the A^R regional technical coefficients matrix to two parts: inter- and intraregional product flows.

$$IIO = \begin{bmatrix} \widehat{\mathrm{T}_{11}^{1}} & \widehat{\mathrm{T}_{12}^{1}} & \dots & \widehat{\mathrm{T}_{1r}^{1}} \\ \widehat{\mathrm{T}_{21}^{1}} & \widehat{\mathrm{T}_{22}^{1}} & \dots & \widehat{\mathrm{T}_{2r}^{1}} \\ \dots & \dots & & \dots \\ \widehat{\mathrm{T}_{r1}^{1}} & \widehat{\mathrm{T}_{r2}^{1}} & \dots & \widehat{\mathrm{T}_{rr}^{1}} \end{bmatrix} \cdot \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \dots & \\ & & & A_r \end{bmatrix} \cdot \begin{bmatrix} X_1 & & & \\ & X_2 & & \\ & & \dots & \\ & & & X_r \end{bmatrix} \qquad (19)$$

$$IIO = \begin{bmatrix} \widehat{\mathrm{T}_{11}^{1}} A_1 X_1 & \widehat{\mathrm{T}_{12}^{1}} A_2 X_2 & \dots & \widehat{\mathrm{T}_{1r}^{1}} A_r X_r \\ \widehat{\mathrm{T}_{21}^{1}} A_1 X_1 & \widehat{\mathrm{T}_{22}^{1}} A_2 X_2 & \dots & \widehat{\mathrm{T}_{2r}^{1}} A_r X_r \\ \dots & \dots & & \dots \\ \widehat{\mathrm{T}_{r1}^{1}} A_1 X_1 & \widehat{\mathrm{T}_{r2}^{1}} A_2 X_2 & \dots & \widehat{\mathrm{T}_{rr}^{1}} A_r X_r \end{bmatrix}, \qquad (20)$$

where $\mathrm{A_r} = \mathrm{Z_r}\widehat{\mathrm{x_r}}^{-1}$ denotes the regional technical coefficients matrix. It shows, which amount of sectoral output from other industries is needed to produce one unit of sectoral output in region r (without defining the input's origin location). While the IIO matrix shows which amount of input is needed from the given and the other region's sectors to produce one unit of sectoral output in a given region. The interregional final use can be generated on the analogy of the operation above.

In the end, we get an interregional input-output table, which can represent the product flows between the region's sectors and final users. However, this matrix is not consistent with the available data at all points, thus in the last step, we need to balance the table to warrant the entire consistency.

Step 5: Balancing the final interregional matrix

Since our full interregional input-output table contained a number of negative cells (changes in inventories, production taxes /subsidies/) traditional RAS method would not work properly. Mainly because it can turn the sum of a given row or column into the opposite sign by adjusting the size of cells. Thus we needed to look for a bi-proportional method that can handle negative values as well. We employed the method called additive RAS developed by Révész (2001).

The logic of the method is the following. First we calculate the share of each cell in each row and column using their absolute values (since they can be negative). Then we distribute the difference between the actual sum of the table (according either to rows or columns, $\sum_j z_{i,j}$) and the desired frame (x_i and x_j).

Step 1 – rows:

$$r_i = \frac{|z_{i,j}^0|}{\sum_j |z_{i,j}^0|} \cdot \left(x_i - \sum_j z_{i,j}^0 \right)$$

$$z_{i,j}^1 = z_{i,j}^1 + r_i$$

Step 2 – columns:

$$r_j = \frac{|z_{i,j}^1|}{\sum_i |z_{i,j}^1|} \cdot \left(x_j - \sum_i z_{i,j}^1 \right)$$

$$z_{i,j}^2 = z_{i,j}^2 + r_j$$

By repeating step 1 and 2 iteratively we can quickly balance the matrix according to the given desired frame. This way we maintain the sign of the table and we make sure that all cells in a given row/column are decreased/increased even if they are negative. An additional advantage of the method is that it can generate an appropriate result even if the sum of a column/row is negative, where the traditional RAS fails to operate (divergence).

2. Description of the spatial general equilibrium model structure

The following chapters contain the description of the multisector CGE model with simplified interregional trade specification. We assume multiple sectors including transportation sector which plays no significant role in this model version because we introduced iceberg transportation cost in a manner that transportation requirements are melted in the process of transportation. We note here that the model may contain two sectors for transportation, one in the previous sense (commodity transport) and the other collecting all other types of transportation (public, business, etc.). The most convenient way of handling these differences, though, would be to move the model onto a make/use basis with different commodities and different sectors also. We do not handle this possibility here. Our terminology in this description is that transportation and production are rendered in *sectors*.

The production is modelled as follows. Labor and capital, in each region, produces a composite production factor or value added according to a CES technology. This composite factor is then used in combination with a composite intermediate good. This composite intermediate good combines different industry inputs according to a Leontief technology (as usual in CGE modeling). Then the composite intermediate and the composite factor are combined with Leontief technology (fixed coefficients). In other words, any industry uses intermediates of all other industries according to fixed coefficients, but these are composite intermediates coming from all regions. The combination of composite intermediates and composite factors provide the local (imports excluded) production of the industry. This regional production activity is then combined with imports to produce a composite output of the regional industries. The composite regional output is split for domestic and export use according to a CET technology.

In this model version good market is defined as a regional market. The output is not supplied in region-region pairs. The interregional trade comes into play by the demand side. The regional output can be used in any regions for different goals (intermediate use, consumption, investment, government consumption) but a given amount of the final product is “melting” in the trade process as a result of iceberg type transportation cost. Interregional trade should satisfy total aggregated demand in the destination region, which can be disaggregated into different final use purposes (consumption, investment, etc.)

In the model description below we use the following notation. Upper case letters are used to denote endogenous variables and lower case letters for parameters. The indices r and q are used to refer to regions whereas indices i and j are used to refer to industries.

Figure 1 and Figure 2 below show the schematic production structure of the model. We also show the types of aggregators used in each step.

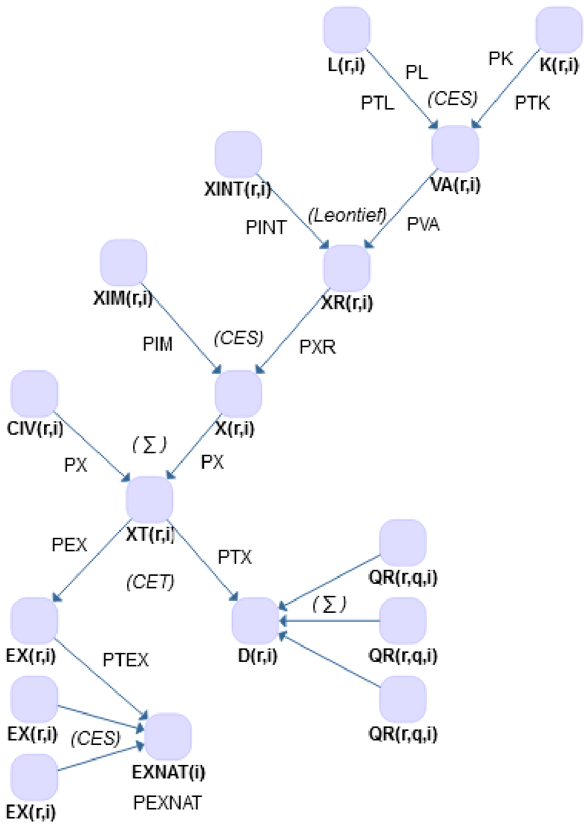


Figure 1: The production structure of the model

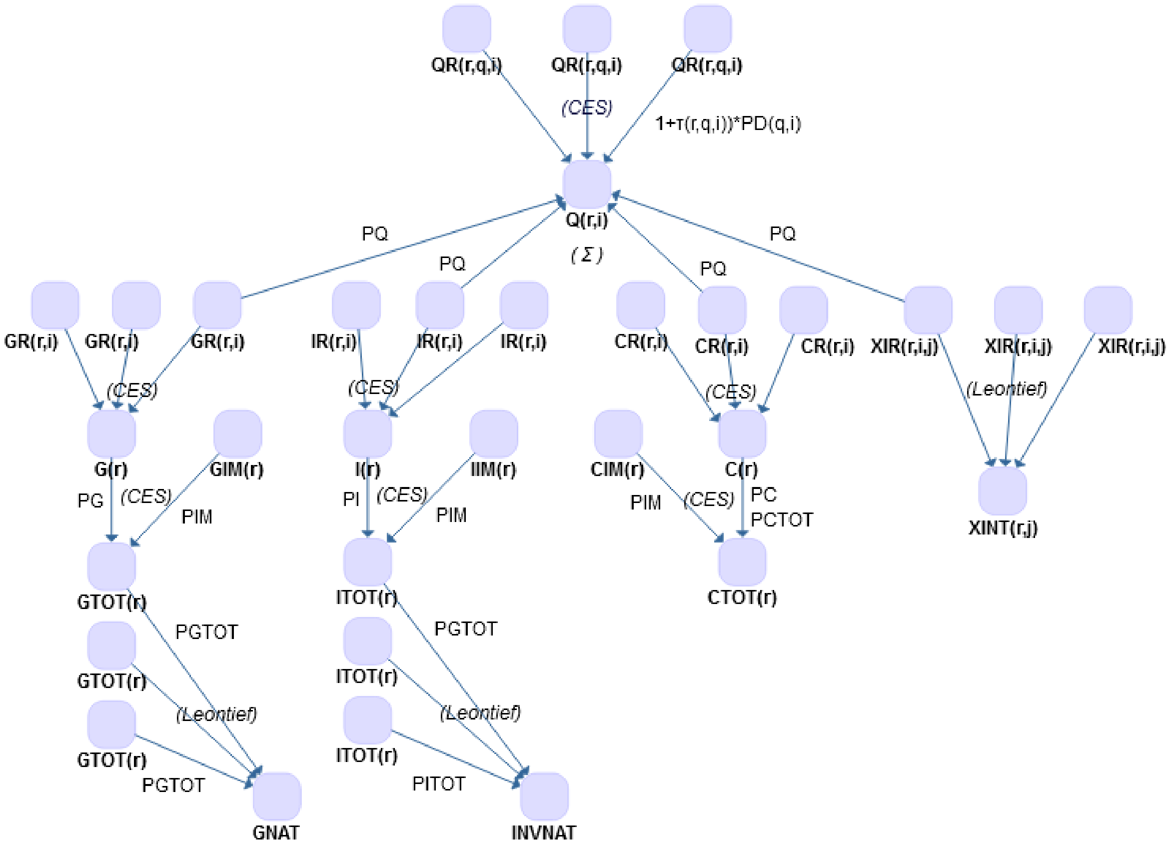


Figure 2: The demand side of the model

Figure 3 explains the income flows of the model, starting from factor incomes and tax revenues and ending at the spending on final goods and savings.

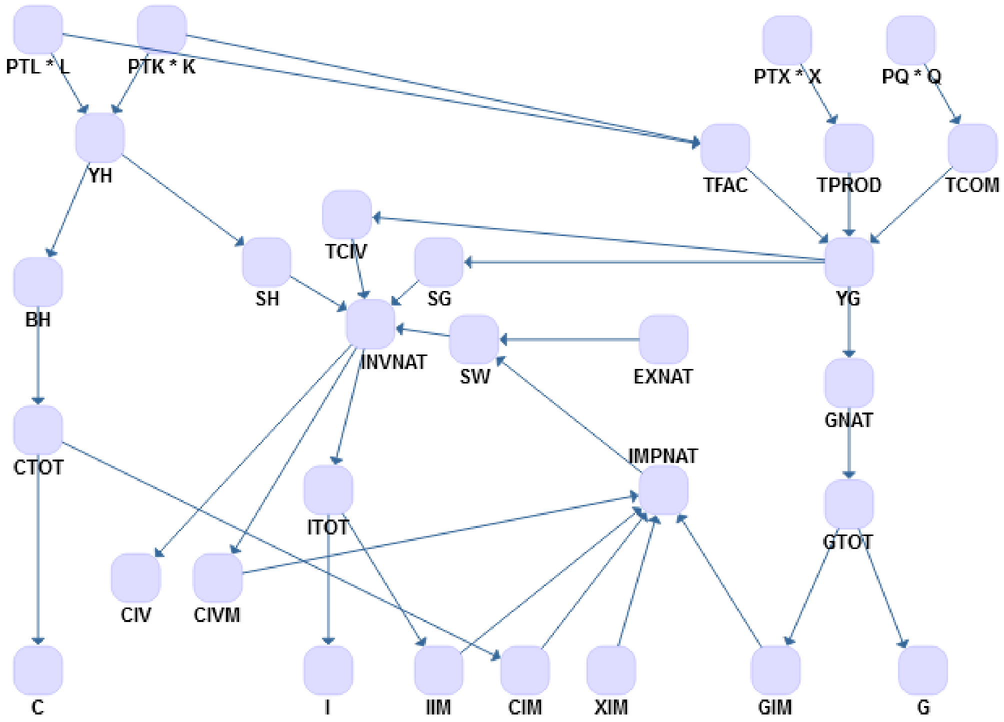


Figure 3: Income flows in the model

2.1. The production of industries

In this section we describe the production structure of the industries (including the transportation sector). For this purpose, we use *j* index to denote all sectors of the economy. An additional index we will introduce is *ac* which stands for all the user agents in the SAM (sectors, households, investment and government /Exports are treated differently/).

2.1.1. Composite factor or value added

The first step of the production combines labor and capital (both specific to the regions) into a composite production factor or value added:

$$VA_{r,j} = d_{r,j}^{VA} \cdot \left[b_{r,j}^{L} \cdot \left(L_{r,j}\right)^{\rho_{r,j}^{VA}} + b_{r,j}^{K} \cdot \left(K_{r,j}\right)^{\rho_{r,j}^{VA}} \right]^{\frac{1}{\rho_{r,j}^{VA}}}$$

where $VA_{r,j}$ is the composite factor used in industry j in region r, $L_{r,j}$ is the labor use and $K_{r,j}$ is the capital use. $b_{r,j}^{L}$ and $b_{r,j}^{K}$ are the share parameters of factors $d_{r,j}^{VA}$ is the TFP while $\rho_{r,j}^{VA}$ is the substitution parameter. From this production function, profit maximization yields the following demand functions for labor and capital:

$$L_{r,j} = \left(\frac{PVA_{r,j}}{PTL_{r,j}}\right)^{\sigma_{r,j}^{VA}} \cdot \left(b_{r,j}^{L}\right)^{\sigma_{r,j}^{VA}} \cdot \left(d_{r,j}^{VA}\right)^{\sigma_{r,j}^{VA}-1} \cdot VA_{r,j}$$

(EQ1 [r,j])

$$K_{r,j} = \left(\frac{PVA_{r,j}}{PTK_{r,j}}\right)^{\sigma_{r,j}^{VA}} \cdot \left(b_{r,j}^{K}\right)^{\sigma_{r,j}^{VA}} \cdot \left(d_{r,j}^{VA}\right)^{\sigma_{r,j}^{VA}-1} \cdot VA_{r,j}$$

(EQ2 [r,j])

where $PVA_{r,j}$ is the price index of the composite factor, $PTL_{r,j}$ and $PTK_{r,j}$ are the unit cost of the labor and capital respectively $\sigma_{r,j}^{VA}$ is the elasticity of substitution which is linked to the substitution parameter: $\rho_{r,j}^{VA} = (\sigma_{r,j}^{VA} - 1)/\sigma_{r,j}^{VA}$. The price index of the value added is defined by the following equation:

$$PVA_{r,j} \cdot VA_{r,j} = PTL_{r,j} \cdot L_{r,j} + PTK_{r,j} \cdot K_{r,j}$$

(EQ3 [r,j])

The relationship between unit costs, taxes included and net prices (taxes excluded are):

$$PTL_{r,j} = (1 + tlab_{r,j}) \cdot PL_r$$

(EQ4 [r,j])

$$PTK_{r,j} = (1 + tcap_{r,j}) \cdot PK$$

(EQ5 [r,j])

where $tlab_{r,j}$ and $tcap_{r,j}$ are the tax rates on labor and capital incomes respectively. Note that tax rates may be region and sector specific: sector specific rates are realistic, region specific rates may ease the calibration. PL_r and PK are the unit labor and capital incomes, taxes excluded. These are specific to the region but not to the industries because factor markets are defined at the regional level.

2.1.2. Local industrial production

The composite factor is merged with intermediates using a Leontief technology. At the first level, we use a composite intermediate combining the intermediates from different regions and different industries:

$$XR_{r,j} = min\left(\frac{1}{v_{r,j}^{VA}} \cdot VA_{r,j}, \frac{1}{v_{r,j}^{XINT}} \cdot XINT_{r,j}\right)$$

where $XR_{r,j}$ is the local production activity of the regional industry (later it will be merged with imports) and $XINT_{r,j}$ is the use of composite intermediates. $v_{r,j}^{VA}$ and $v_{r,j}^{XINT}$ are the coefficients defining the requirement of composite factors and composite intermediates in order to produce one unit of output. The demand functions for composites factors and intermediates are as follows:

$$VA_{r,j} = v_{r,j}^{VA} \cdot XR_{r,j}$$

(EQ6 [r,j])

$$XINT_{r,j} = v_{r,j}^{XINT} \cdot XR_{r,j}$$

(EQ7 [r,j])

The price index of the local regional output ($PXR_{r,i}$) is defined by the following equation:

$$PXR_{r,j} \cdot XR_{r,j} = PVA_{r,j} \cdot VA_{r,j} + PINT_{r,j} \cdot XINT_{r,j}$$

(EQ8 [r,j])

where $PINT_{r,j}$ is the price index of the composite intermediate good (to be defined later).

2.1.3. Regional industry output

The local production activity $XR_{r,j}$ is merged with imports through a CES aggregator, resulting in regional industry output $X_{r,j}$:

$$X_{r,j} = d_{r,j}^{X1} \cdot \left[b_{r,j}^{XR} \cdot \left(XR_{r,j}\right)^{\rho_{r,j}^{X1}} + b_{r,j}^{XIM} \cdot \left(XIM_{r,j}\right)^{\rho_{r,j}^{X1}} \right]^{\frac{1}{\rho_{r,j}^{X1}}}$$

where $X_{r,j}$ is the composite regional output, $XR_{r,j}$ is the local production activity and $XIM_{r,j}$ is the import (import is not broken down to commodities, j here refers to the industry producing). $b_{r,j}^{XR}$ and $b_{r,j}^{XIM}$ are the share parameters, $d_{r,j}^{X1}$ is the shift parameter, while $\rho_{r,j}^{X1}$ is the substitution parameter. From this production function, profit maximization yields the following demand functions:

$$XR_{r,j} = \left(\frac{PX_{r,j}}{PXR_{r,j}}\right)^{\sigma_{r,j}^{X1}} \cdot \left(b_{r,j}^{XR}\right)^{\sigma_{r,j}^{X1}} \cdot \left(d_{r,j}^{X1}\right)^{\sigma_{r,j}^{X1}-1} \cdot X_{r,j}$$

(EQ9 [r,j])

$$XIM_{r,j} = \left(\frac{PX_{r,j}}{PIM_{r}}\right)^{\sigma_{r,j}^{X1}} \cdot \left(b_{r,j}^{XIM}\right)^{\sigma_{r,j}^{X1}} \cdot \left(d_{r,j}^{X1}\right)^{\sigma_{r,j}^{X1}-1} \cdot X_{r,j}$$

(EQ10 [r,j])

where $PX_{r,j}$ is the price index of the regional output (producer price, production taxes not included), $PXR_{r,j}$ and PIM_r are the price indices of the local industrial activity and import respectively, $\sigma_{r,j}^{X1}$ is the elasticity of substitution which is linked to the substitution parameter: $\rho_{r,j}^{X1} = (\sigma_{r,j}^{X1} - 1)/\sigma_{r,j}^{X1}$. The price index of the value added is defined by the following equation:

$$PX_{r,j} \cdot X_{r,j} = PXR_{r,j} \cdot XR_{r,j} + PIM_r \cdot XIM_{r,j}$$

(EQ11 [r,j])

Note, that we allow for different import prices to prevail in different regions and also different import prices in different sectors. This may reflect transportation costs, but the prices may be set equal across regions and across sectors if necessary.

Changes in inventories can vary between negative and positive values it can completely change the sign of total regional final demand and thus interregional trade which can create problems for the model since standard CES and CET function are not working properly with negative quantities. Thus we account for negative changes in inventories as additional output in time period t and positive changes in inventories as a decrease in current output.

$$XT_{r,j} = X_{r,j} + CIV_{r,j}$$

Where $XT_{r,j}$ can be considered as total composite regional output in time period t (including current production and sales from previous inventories, but not containing current increase of inventories).

2.1.4. Transformation of output into supply

Once total output is produced it is first split between export and domestic supplies according to a CET technology:

$$XT_{r,j} = d_{r,j}^{X2} \cdot \left[b_{r,j}^{EX} \cdot \left(EX_{r,j}\right)^{\rho_{r,j}^{X2}} + b_{r,j}^{D} \cdot \left(D_{r,j}\right)^{\rho_{r,j}^{X2}} \right]^{\frac{1}{\rho_{r,j}^{X2}}}$$

where $EX_{r,j}$ is the exported quantity, $D_{r,j}$ is the domestic quantity. $d_{r,j}^{X2}$ is the shift parameter whereas $b_{r,j}^{EX}$ and $b_{r,j}^{D}$ are the respective share parameters. $\rho_{r,j}^{X2}$ is the transformation parameter. From this CET function, the supply for export and domestic use can be derived as follows:

$$EX_{r,j} = \left(\frac{PEX_{r,j}}{PTX_{r,j}}\right)^{\sigma_{r,j}^{X2}} \cdot \left(b_{r,j}^{EX}\right)^{-\sigma_{r,j}^{X2}} \cdot \left(d_{r,j}^{X2}\right)^{-\sigma_{r,j}^{X2}-1} \cdot XT_{r,j}$$

(EQ12 [r,j])

Next we separated normal industries and the transportation sector. The regional supply of industry output is determined by the next equation:

$$D_{r,j} = \left(\frac{PD_{r,j}}{PTX_{r,j}}\right)^{\sigma_{r,j}^{X2}} \cdot \left(b_{r,j}^{D}\right)^{-\sigma_{r,j}^{X2}} \cdot \left(d_{r,j}^{X2}\right)^{-\sigma_{r,j}^{X2}-1} \cdot XT_{r,j}$$

(EQ13 [r,j])

where $PEX_{r,j}$ is the export price index, $PD_{r,j}$ is the price index of the regional supply for domestic use and $PTX_{r,j}$ is the output price with production taxes included. $\sigma_{r,j}^{X2}$ is the elasticity of transformation, determined by the transformation parameter: $\sigma_{r,j}^{X2} = 1/(\rho_{r,j}^{X2} - 1)$. The zero profit conditions for this stage of the production is:

$$PTX_{r,j} \cdot XT_{r,j} = PEX_{r,j} \cdot EX_{r,j} + PD_{r,j} \cdot D_{r,j}$$

(EQ14 [r,j])

The relationship between pre- and post-tax prices is simply defined as:

$$PTX_{r,j} = (1 + tprod_{r,j}) \cdot PX_{r,j}$$

(EQ15 [r,j])

where $tprod_{r,j}$ is the region- and sector-specific production tax rate.

2.2. Interregional trade and transportation

This is the final step of production and these composite products are used for consumption (final and intermediate). However, iceberg transportation costs are included in the prices of interregional trade prices which will bring spatial relationships in the prices. In this case the difference between FOB and CIF prices can be viewed as the transportation cost $(1 + \tau_{r,q,i})$. In this setting $\tau_{r,q,i}$ will determine the transportation cost between region *r* and *q* in case of an interregional import regarding industrial product *i*.

Total interregional trade is captured by $QR_{q,r,i}$ which shows us the total amount of goods and services transported between two regions. Later these products will be consumes/used by different actors in the destination region (e.g. households, government, etc.) The interregional trade is driven by the total amount of regional demand ($Q_{r,i}$ which will be discussed later) through a CES function. We do not explicitly model the interregional supply side. We assume that the regional demand will be satisfied from all of the regions through an equilibrium condition.

$$Q_{r,i} = d_{r,i}^{Q} \cdot \left[\sum_{q} b_{q,r,i}^{Q} \cdot \left(\frac{QR_{q,r,i}}{(1 + \tau_{q,r,i})} \right)^{\rho_{r,i}^{Q}} \right]^{\frac{1}{\rho_{r,i}^{Q}}}$$

$$\frac{QR_{q,r,i}}{1 + \tau_{q,r,i}} = \left(\frac{PQ_{r,i}}{(1 + \tau_{q,r,i}) \cdot PD_{q,i}} \right)^{\sigma_{r,i}^{Q}} \cdot \left(b_{q,r,i}^{Q}\right)^{\sigma_{r,i}^{Q}} \cdot \left(d_{r,i}^{Q}\right)^{\sigma_{r,i}^{Q} - 1} \cdot Q_{r,i}$$

(EQ16[q,r,i])

$$\sum_{q} (1 + \tau_{q,r,i}) \cdot PD_{q,i} \cdot \frac{QR_{q,r,i}}{1 + \tau_{q,r,i}} = PQ_{r,i} \cdot Q_{r,i}$$

(EQ17[r,i])

2.2.1. Commodity tax on regional demand

Additionally commodity taxes are calculated on these sales. As a general case, we allow for specific commodity tax rates for all final users: e.g. households in region r may pay different tax rates on commodities than the tax rates paid by firms for instance. The general formulation allows for any kind of aggregation (e.g. we may simplify to have one single tax rate over the economy). The main reason for this formulation is that SAM data used for model calibration may supply commodity tax expenses separately for all final users and their spendings for different products. Let us denote the commodity tax rate of household spending, government spending, investment, industry intermediates, industry export respectively by $tcomCR_{r,i}$, $tcomGR_{r,i}$, $tcomIR_{r,i}$, $tcomXIR_{r,i}$, $tcomEX_{r,i}$, for purchases by users in region r. The respective post-tax prices are $PCR_{r,i}$, $PGR_{r,i}$, $PIR_{r,i}$, $PXIR_{r,i,j}$, $PTEX_{r,j}$. Then, the relationships between pre- and post-tax prices are given by the following relationships:

$$PCR_{r,i} = (1 + tcomCR_{r,i}) \cdot PQ_{r,i}$$

(EQ18 [r,i])

$$PGR_{r,i} = (1 + tcomGR_{r,i}) \cdot PQ_{r,i}$$

(EQ19 [r,i])

$$PIR_{r,i} = (1 + tcomIR_{r,i}) \cdot PQ_{r,i}$$

(EQ20 [r,i])

$$PXIR_{r,i,j} = (1 + tcomXIR_{r,j}) \cdot PQ_{r,i}$$

(EQ21 [r,i,j])

$$PTEX_{r,j} = (1 + tcomEX_{r,j}) \cdot PEX_{r,j}$$

(EQ22 [r,j])

As export demand is directly determined against the output of the different regions, export tax can be only interpreted separately for the source regions in the domestic economy.

2.3. Capital supply

Households own factors of production. They can freely decide where to allocate their resources. In case of labour (as you can see it later) this adjustment is more rigid and slower than in case of capital. We assume that households can reallocate their capital resources between regions based on regional differences in capital prices. This regional capital supply will be set by a CET function where the regional capital supply is given as follows:

$$KN = d^{KR} \cdot \sum_{r} \left(b_r^{KR} \cdot {KR_r}^{\rho^{KR}} \right)^{\frac{1}{\rho^{KR}}}$$

Where the regional capital supply can be derived as follows:

$$KR_r = \left(\frac{PKR_r}{PKN} \right)^{\sigma^{KR}} \cdot (b_r^{KR})^{-\sigma^{KR}} \cdot (d^{KR})^{-\sigma^{KR}-1} \cdot KN$$

(EQ23[r])

Where d^{KR} is the shift parameter of the CET function and b_r^{KR} refers to the share parameter. The zero-profit condition must hold for the national level capital market as well:

$$\sum_r PKR_r \cdot KR_r = PKN \cdot KN$$

(EQ24[1])

Where KN is the national capital stock and KR_r is the regional capital supply. Their prices respectively are PKR_r and PKN.

The equilibrium condition for regional capital marke takes the following form:

$$KN = \sum_r KS_r$$

(EQ25[1])

Where KS_r is the regional capital stock owned by households. The equation above implies that households are payed after their capital supply by national capital prices (PKN) which is selected as numeraire. Thus equation 28 is not included in the model since according to the Walras law it is considered to be redundant.

Than in a next step regional capital supply can be distributed among different activities. It can be governed by market automatisms in an equilibrium condition with an implicit assumption that capital is perfectly mobile between activities. Or it can be governed by a CET function which will determine 'imperfectly' the distribution of sectoral capital supply, based on price differences:

$$KR_r = d_r^{KR} \cdot \sum_i \left(b_{r,i}^{KI} \cdot {KI_{r,i}}^{\rho^{KI}}\right)^{\rho^{KI}}$$

Industry level capital supply thus can be written as follows:

$$KI_{r,i} = \left(\frac{PTK_{r,i}}{PKR_r}\right)^{\sigma_r^{KI}} \cdot \left(b_{r,i}^{KI}\right)^{-\sigma_r^{KI}} \cdot (d_r^{KR})^{-\sigma_r^{KI}-1} \cdot KR_r$$

(EQ26[r,i])

Where d_r^{KR} is the shift parameter of industry level regional capital supply CET function and $b_{r,i}^{KI}$ refers to the share parameter of industrial capital supply. The zero-profit condition in this case takes the following form:

$$\sum_i PTK_{r,i} \cdot KI_{r,i} = PKR_r \cdot KR_r$$

(EQ27[r])

Where $KI_{r,i}$ is the sectoral supply of regional capital with the same price as capital demand: $PTK_{r,i}$.

Equilibrium condition for industry level capital market:

$$K_{r,i} = KI_{r,i}$$

(EQ28[r,i])

2.4. Incomes and savings

2.4.1. Households

Household income is defined on the regional level. Total income of the regional households is made up of labor and capital incomes generated in the regional industry sectors:

$$YH_r = PL_r \cdot LS_r + PKN \cdot KS_r$$

(EQ23 [r])

where YH_r is the total income of the households. Households are assumed to save a constant fraction (sy_r) of their income, so household saving (SH_r) is:

$$SH_r = sy_r \cdot YH_r$$

(EQ24 [r])

Accordingly, households' consumption budget (BH_r) is defined simply as:

$$BH_r = (1 - sy_r) \cdot YH_r$$

(EQ25 [r])

2.4.2. Government

Government is assumed to have revenues from three sources: production tax, commodity (or product) tax and income tax (on factors). For all three taxes we define a region- and commodity/industry-specific exogenous tax-rate, then the revenue from production tax ($TAXPROD$) can be written as:

$$TAXPROD = \sum_r \sum_j tprod_{r,j} \cdot PX_{r,j} \cdot XT_{r,j}$$

(EQ26 [1])

where $tprod_{r,jt}$ are the tax rates in the industries. The income tax on factors is defined as:

$$TAXFAC = \sum_r \sum_j tlab_{r,j} \cdot PL_r \cdot L_{r,j} + \sum_r \sum_j tcap_{r,j} \cdot PK_r \cdot K_{r,j}$$

(EQ27 [1])

where $tlab_r$ and $tcap_r$ are the labor and capital income tax rates in the industries. Commodity taxes are defined as

$$TAXCOM = \sum_r \sum_i \sum_j (tcomXIR_{r,i} \cdot PQ_{r,i} \cdot XIR_{r,i,j}) + \sum_r \sum_i (tcomCR_{r,i} \cdot PQ_{r,i} \cdot CR_{r,i} + tcomIR_{r,i} \cdot PQ_{r,i} \cdot IR_{r,i} + tcomGR_{r,i} \cdot PQ_{r,i} \cdot GR_{r,i}) + \sum_r \sum_i tcomEX_{r,i} \cdot PEX_{r,i} \cdot EX_{r,i} - \sum_r \sum_i tcomCIV_{r,i} \cdot PX_{r,i} \cdot CIV_{r,i}$$

(EQ28 [1])

Taxes on inventories are also added to tax revenues. Although inventory tax rates are negative we still need to account for them in government revenues (as certain expenditures, subsidies). Eventually these amounts will increase the total available investment fund in the economy. If tax rates would be positive, they would decrease total savings and increase government revenues.

Government income is the sum of taxes:

$$YG = (TAXPROD + TAXFAC + TAXCOM)$$

(EQ29 [1])

Government expenditure is defined as the difference between tax revenues and government deficit:

$$GNAT = YG - SG$$

(EQ30 [1])

2.5. Demand

2.5.1. Intermediate demand

In the model we have two distinct branches of production which use intermediates goods to produce output: industries and the transportation sector. As described before, industries use $XINT_{r,j}$ composite intermediate goods. As in standard models, composite intermediate good used by industry j is a Leontief aggregate of inputs produced by other industries:

$$XINT_{r,j} = min\left(\frac{1}{a_{r,i,1}} \cdot XIR_{r,i,1}, \frac{1}{a_{r,i,2}} \cdot XIR_{r,i,2}, \dots, \frac{1}{a_{r,i,j}} \cdot XIR_{r,i,j}\right)$$

where $XIR_{r,i,j}$ is the volume of industry input i used by industry j in region r. $a_{r,i,j}$ is the standard input-output coefficient reflecting the amount of product i required by industry j in region r in order to have one unit of the intermediate composite $XINT_{r,j}$. The resulting demand functions are:

$$XIR_{r,i,j} = a_{r,i,j} \cdot XINT_{r,j}$$

(EQ31 [r,i,j])

This aggregation is closed by the value equation determining the price index of the composite intermediate $PINT_{r,j}$:

$$PINT_{r,j} \cdot XINT_{r,j} = \sum_i PXIR_{r,i,j} \cdot XIR_{r,i,j}$$

(EQ32 [r,j])

2.5.2. Consumption demand

On the first level, households consume a composite consumption good, the price level of which is defined by the following equation:

$$PCTOT_r \cdot CTOT_r = BH_r$$

(EQ33 [r])

where $CTOT_r$ is the composite (real) consumption of households and $PCTOT_r$ is its price index. Composite consumption is composed of domestic and imported goods under the following CES technology:

$$CTOT_r = d_r^{CTOT} \cdot \left[b_r^C \cdot (C_r)^{\rho_r^{CTOT}} + b_r^{CIM} \cdot (CIM_r)^{\rho_r^{CTOT}}\right]^{\frac{1}{\rho_r^{CTOT}}}$$

where C_r is consumption from domestic sources, CIM_r is consumption from imports, d_r^{CTOT} is the shift parameter, b_r^C and b_r^{CIM} are the share parameters and ρ_r^{CTOT} is the substitution parameter. This aggregator gives rise to the following demand functions:

$$C_r = \left(\frac{PCTOT_r}{PC_r}\right)^{\sigma_r^{CTOT}} \cdot (b_r^C)^{\sigma_r^{CTOT}} \cdot (d_r^{CTOT})^{\sigma_r^{CTOT}-1} \cdot CTOT_r$$

(EQ34 [r])

$$CIM_r = \left(\frac{PCTOT_r}{PIM_r}\right)^{\sigma_r^{CTOT}} \cdot (b_r^{CIM})^{\sigma_r^{CTOT}} \cdot (d_r^{CTOT})^{\sigma_r^{CTOT}-1} \cdot CTOT_r$$

(EQ35 [r])

where PC_r and PIM_r are the domestic and import price indices. This level of consumption is closed with the value equation:

$$PCTOT_r \cdot CTOT_r = PC_r \cdot C_r + PIM_r \cdot CIM_r$$

(EQ36 [r])

Domestic composite consumption, on the other hand is a CES aggregate of different industrial products:

$$C_r = d_r^C \cdot \left[\sum\nolimits_i b_{r,i}^{CR} \cdot \left(CR_{r,i}\right)^{\rho_r^C}\right]^{\frac{1}{\rho_r^C}}$$

where $CR_{r,i}$ is the consumption of households in region r of product i. $b_{r,i}^{CR}$ are the share parameters, d_r^C is the shift parameter and ρ_r^C is the substitution parameter. The consumption demand functions from this aggregator are as follows:

$$CR_{r,i} = \left(\frac{PC_r}{PCR_{r,i}}\right)^{\sigma_r^C} \cdot \left(b_{r,i}^{CR}\right)^{\sigma_r^C} \cdot (d_r^C)^{\sigma_r^C - 1} \cdot C_r$$

(EQ37 [r,i])

The corresponding value equation is:

$$PC_r \cdot C_r = \sum_i PCR_{r,i} \cdot CR_{r,i}$$

(EQ38 [r])

2.5.3. Investment demand

Total national investment in nominal terms is denoted by $INVNAT$. Total national investment is allocated to regional investment budgets according to fixed shares denoted by si_r:

$$PITOT_r \cdot ITOT_r = si_r \cdot \left(INVNAT + \sum_{r,i}(1 + tcomCIV_{r,i}) \cdot PX_{r,i} \cdot CIV_{r,i} - \sum_r PIM \cdot CIVM_r\right)$$

(EQ39 [r])

where $ITOT_r$ is total real investment in a region while $PITOT_r$ is its price index. At this point we have to emphasize that this equation works only with the current setting of our social accounting matrix. Each element in CIV below zero means increase in inventories, and positive elements represents decrease (sales). Thus sales mean additional source for investment, while increased investment means a loss of potential investment (in the current period), these additional investments has to be produced and the production must be financed by savings so it will decrease the amount of available investment funds. However inventory import is treated differently. Positive CIVM means increase, a negative means decrease in inventories, so we need to subtract them from investment funds.

As with consumption, $ITOT_r$ is a composite of investment goods purchased from domestic sources and import according to a CES aggregator:

$$ITOT_r = d_r^{ITOT} \cdot \left[b_r^I \cdot (I_r)^{\rho_r^{ITOT}} + b_r^{IIM} \cdot (IIM_r)^{\rho_r^{ITOT}}\right]^{\frac{1}{\rho_r^{ITOT}}}$$

where I_r is investment from domestic sources, IIM_r is investment from imports, d_r^{ITOT} is the shift parameter, b_r^I and b_r^{IIM} are the share parameters and ρ_r^{ITOT} is the substitution parameter. This aggregator gives rise to the following demand functions:

$$I_r = \left(\frac{PITOT_r}{PI_r}\right)^{\sigma_r^{ITOT}} \cdot (b_r^I)^{\sigma_r^{ITOT}} \cdot (d_r^{ITOT})^{\sigma_r^{ITOT} - 1} \cdot ITOT_r$$

(EQ40 [r])

$$IIM_r = \left(\frac{PITOT_r}{PIM_r}\right)^{\sigma_r^{ITOT}} \cdot (b_r^{IIM})^{\sigma_r^{ITOT}} \cdot (d_r^{ITOT})^{\sigma_r^{ITOT} - 1} \cdot ITOT_r$$

(EQ41 [r])

where PI_r and PIM_r are the domestic and import price indices. This level of consumption is closed with the value equation:

$$PITOT_r \cdot ITOT_r = PI_r \cdot I_r + PIM_r \cdot IIM_r$$

(EQ42 [r])

Domestic investment, on the other hand is a CES aggregate of different industry products:

$$I_r = d_r^I \cdot \left[\sum_i b_{r,i}^{IR} \cdot (IR_{r,i})^{\rho_r^I}\right]^{\frac{1}{\rho_r^I}}$$

where $IR_{r,i}$ is the investment in region r using product i. $b_{r,i}^{IR}$ are the share parameters, d_r^I is the shift parameter and ρ_r^I is the substitution parameter. The investment demand functions from this aggregator are as follows:

$$IR_{r,i} = \left(\frac{PI_r}{PIR_{r,i}}\right)^{\sigma_r^I} \cdot (b_{r,i}^{IR})^{\sigma_r^I} \cdot (d_r^I)^{\sigma_r^I-1} \cdot I_r$$

(EQ43 [r,i])

The corresponding value equation is:

$$PI_r \cdot I_r = \sum_i PIIR_{r,i} \cdot IIR_{r,i}$$

(EQ44 [r,i])

2.5.4. Government demand

The demand of government follows similar rules to consumption and investment. Total nominal government expenditure is denoted by $GNAT$. In order to fit into the region-region structure of the model, these expenditures allocated to regions according to exogenous region shares:

$$PGTOT_r \cdot GTOT_r = sg_r \cdot GNAT$$

(EQ45 [r])

This rule allows for a reallocation of government funds to local governments but also allows keeping all the expenditures in a central region. Total real government expenditures in a region are a composite of domestic and import goods as defined by the following CES technology:

$$GTOT_r = d_r^{GTOT} \cdot \left[b_r^G \cdot (G_r)^{\rho_r^{GTOT}} + b_r^{GIM} \cdot (GIM_r)^{\rho_r^{GTOT}}\right]^{\frac{1}{\rho_r^{GTOT}}}$$

where G_r is government purchases from domestic sources, GIM_r is government purchases from imports, d_r^{GTOT} is the shift parameter, b_r^G and b_r^{GIM} are the share parameters and ρ_r^{GTOT} is the substitution parameter. This aggregator gives rise to the following demand functions:

$$G_r = \left(\frac{PGTOT_r}{PG_r}\right)^{\sigma_r^{GTOT}} \cdot (b_r^G)^{\sigma_r^{GTOT}} \cdot (d_r^{GTOT})^{\sigma_r^{GTOT}-1} \cdot GTOT_r$$

(EQ46 [r])

$$GIM_r = \left(\frac{PGTOT_r}{PIM_r}\right)^{\sigma_r^{GTOT}} \cdot \left(b_r^{GIM}\right)^{\sigma_r^{GTOT}} \cdot \left(d_r^{GTOT}\right)^{\sigma_r^{GTOT}-1} \cdot GTOT_r$$

(EQ47 [r])

where PG_r and PIM_r are the domestic and import price indices. This level of government spending is closed with the value equation:

$$PGTOT_r \cdot GTOT_r = PG_r \cdot G_r + PIM_r \cdot GIM_r$$

(EQ48 [r])

Government purchases from domestic sources, on the other hand, are composed of different products according to the CES aggregator:

$$G_r = d_r^G \cdot \left[\sum\nolimits_i b_{r,i}^{GR} \cdot \left(GR_{r,i}\right)^{\rho_r^G}\right]^{\frac{1}{\rho_r^G}}$$

where $GR_{r,i}$ is the amount of industry product i bought by the government in region r from region q. $b_{r,i}^{GR}$ are the share parameters, d_r^G is the shift parameter and ρ_r^G is the substitution parameter. The consumption demand functions from this aggregator are as follows:

$$GR_{r,i} = \left(\frac{PG_r}{PGR_{r,i}}\right)^{\sigma_r^G} \cdot \left(b_{r,i}^{GR}\right)^{\sigma_r^G} \cdot \left(d_r^G\right)^{\sigma_r^G-1} \cdot G_r$$

(EQ49 [r,i])

The corresponding value equation is:

$$PG_r \cdot G_r = \sum\nolimits_i PGR_{r,i} \cdot GR_{r,i}$$

(EQ50 [r,i])

2.5.5. Export demand

Export demand for industry output is defined through two aggregated variables: real demand for exported goods towards the whole country ($EXNAT_j$) and its price index ($PEXNAT_j$). The real export demand is a CES composite of exported goods from different regions according to the following aggregator:

$$EXNAT_j = d_j^{EXNAT} \cdot \left[\sum\nolimits_r b_{r,j}^{EXR} \cdot \left(EX_{r,j}\right)^{\rho_j^{EXNAT}}\right]^{\frac{1}{\rho_j^{EXNAT}}}$$

where $EX_{r,j}$ is the amount of industry product j bought on the export market from region r. $b_{r,j}^{EXR}$ are the share parameters, d_j^{EXNAT} is the shift parameter and ρ_j^{EXNAT} is the substitution parameter. This aggregator gives rise to the following demand functions for regional export:

$$EX_{r,j} = \left(\frac{PEXNAT_j}{PTEX_{r,j}}\right)^{\sigma_j^{EXNAT}} \cdot \left(b_{r,j}^{EXR}\right)^{\sigma_j^{EXNAT}} \cdot \left(d_j^{EXNAT}\right)^{\sigma_j^{EXNAT}-1} \cdot EXNAT_j$$

(EQ51 [r,j])

where σ_j^{EXNAT} is the substitution elasticity, linked to the substitution parameter: $\rho_j^{EXNAT} = (\sigma_j^{EXNAT} - 1)/\sigma_j^{EXNAT}$. The following value equation closes the export demand:

$$PEXNAT_j \cdot EXNAT_j = \sum_r PTEX_{r,j} \cdot EX_{r,j}$$

(EQ52 [j])

Note, that we should have a different notation for regional export demands from EX_r which was introduced for export supply. However, in this case we should have additional equilibrium conditions which belong to the foreign sector and it would unnecessarily complicate the model, as long as the export is treated exogenously.

There should be an option to link industry exports in a same manner than in EQ17. In the present model setting we disregard this option.

2.6. Equilibrium conditions

The model is closed by the equilibrium conditions defined below.

First, equilibrium on the goods market is established on a region-region specific basis:

$$CR_{r,i} + IR_{r,i} + GR_{r,i} + \sum_j XIR_{r,i,j} = Q_{r,i}$$

(EQ53 [r,i])

Instead of using a CET function to distribute regional supply ($D_{r,i}$) to other destination regions we use another equilibrium condition which simplifies the model even more. This way we dropped our initial assumption of relational markets, in this formulation we assumed regional markets. This means also that the price index of $D_{r,i}$ and $QR_{r,q,i}$ is the same $PD_{r,i}$ (FOB price).

$$D_{r,i} = \sum_q QR_{r,q,i}$$

(EQ54 [r,i])

Equilibrium on the regional factor markets equates labor and capital supplies (given exogenously) and demand in the industries.

$$\sum_j L_{r,j} = ls_r$$

(EQ55 [r])

$$\sum_{r,j} K_{r,j} = \sum_{r} ks_r$$

(EQ56 [r])

We defined national capital market and we set the price of capital as numeriare of the model.

Let’s define the foreign saving (current account balance in foreign currency) as follows:

$$SW = \sum_r PWM_r \cdot \left(\sum_j (XIM_{r,j}) + CIM_r + IIM_r + GIM_r + CIVM_r \right) - \sum_j PWE_j \cdot EXNAT_j$$

(EQ57 [1])

Finally, total savings (saving of households, the government and the foreign sector) must be equal to investment:

$$INVNAT = \sum_r SH_r + SW \cdot er + SG$$

(EQ58 [1])

Where *SW* refers to foreign savings in foreign currency and *er* refers to the exchange rate.

With respect to exports, we have the choice to fix either export price level or export quantity. Once one of these is fixed, EQ45 defines the other. Our choice here is to leave quantities to be determined, provide an exogenous world export price and determine the export price in home currency according to this:

$$PEXNAT_j = PWE_j \cdot er$$

(EQ59 [j])

where $pexnatp$ is a parameter fixing world export prices.

Import prices must also be fixed without the specification of a foreign supply scheme:

$$PIM_r = PWM_r \cdot er$$

(EQ60 [r])

Government saving is set by the deficit per GDP as a portion of current GDP.

$$SG = ed \cdot adj \cdot \sum_r \sum_i (PVA_{r,i} \cdot VA_{r,i})$$

(EQ61 [1])

Where the value of *ed* (deficit per GDP) is updated in each time period by the macro block according to a restrictive deficit rule (discussed in a later chapter).

2.7. Model dynamics

The dynamic behavior of the model is introduced in two steps. First, we will describe the process of capital accumulation then we will introduce the interregional migration of labor. These two dynamic aspects of the model is 'separated' from the model itself. They are partially determined outside the model after each time period. Since the model is recursive, each period can be considered as a static model and these static steps are connected through migration and capital accumulation.

2.7.1. Capital accumulation

The accumulation of capital is determined by a standard equation:

$$KS_{r,t+1} = (1-\delta) \cdot KS_{r,t} + \beta \cdot \frac{ITOT_{r,t}}{PK_t} \tag{C1}$$

Where $KS_{r,t}$ is the capital supply in region r and time period t. δ is the depreciation rate, β is the rate of conversion between capital supply and investment. This step is needed due to the problems we faced before. The total saving should increase the national capital supply but we have information about capital use from the interregional SAM. Thus we convert the level of investment to capital use using a parameter. Where β is determined by the following equation:

$$\beta = \frac{KS0}{CS} \tag{C2}$$

Where KS0 is the initial total capital use from the original SAM and CS is the total national capital stock from the Hungarian Statistical Office. The capital supply has role only in the regional income determination because we assumed previously that we have one national capital market. Thus capital is completely mobile between regions but the regional capital income is determined in equation 23. The value of β is set in a way that regional income growth rate will fit the available rate of statistical data.

2.7.2. Migration

We assumed that only interregional migration determines the regional labor supply since we will ignore demographic changes. In the model, migration itself is considered as net regional migration which is determined by the following equation which is based on the difference between regional utility ($U_{r,t}$) and the average national utility per capita:

$$LMIGR_{r,t} = LS_{r,t} \cdot \varphi \cdot \left(e^{\theta \cdot U_{r,t}} - e^{\theta \cdot \frac{\sum_r U_{r,t} \cdot LS_{r,t}}{\sum_r LS_{r,t}}} \right) \tag{L1}$$

Where φ and θ are sensitivity parameters and their values are borrowed from GMR-Hungary. The utility function is based on consumption ($CTOT_r$) per capita and housing (H_r) per capita:

$$U_{r,t} = initC_r + \alpha H \cdot \ln\left(\frac{H_{r,t}}{N_{r,t}}\right) + \beta H \cdot \ln\left(\frac{CTOT_{r,t}}{N_{r,t}}\right) \tag{L2}$$

Where *initC*$_r$, *αH* and *βH* are parameters that should be calibrated. The value of *αH* and β*H* is borrowed from the original GMR-Hungary model, *initC*$_r$ is calibrated in in order to achieve complete consistency between $LMIGR_{r,t}$ (t=1) and actual net domestic migration data from 2010. Next we can determined the change of regional labor supply:

$$LS_{r,t+1} = LS_{r,t} + LMIGR_{r,t} \tag{L3}$$

After that we need to update the regional population. We assume that as labor supply is changed by migration, population is changed accordingly. The equation again is borrowed from the GMR-Hungary and it is based on the difference between net regional migration and the average migration multiplied by the fixed conversion ratio between labor and capital:

$$NMIGR_{r,t} = \left(LMIGR_{r,t} - LS_{r,t} \cdot \frac{\sum_r LMIGR_{r,t}}{\sum_r LS_{r,t}}\right) \cdot \frac{\sum_r N_{r,t}}{\sum_r LS_{r,t}} \tag{L4}$$

Finally, the change of population can be described in a similar way to labor migration:

$$N_{r,t+1} = N_{r,t} + NMIGR_{r,t} \tag{L5}$$

2.7.3. TFP model block

This section describes the change of total factor productivity of industries in different regions. The model itself is borrowed from previous GMR modeling approach. The external TFP equations can be ran separately from the model, the model itself contains only one TFP equation which will interact with all the other blocks.

The change of TFP is set by the following equations:

$$\begin{aligned}\ln(TFP_{r,t+1}) = {} & tfp\_const_r + fedum\_coef_{r,t} \cdot FEDUM_{r,t} + vadum_{coef_{r,t}} \cdot VADUM_{r,t} \\ & + eduv\_coef_{r,t} \cdot (\ln(EDUV_{r,t}) + \ln(EDUSTOCK_{r,t})) \cdot \ln(PSTCKRT_{r,t}) \\ & + emp\_coef_{r,t} \cdot \ln(LS_{r,t})\end{aligned} \tag{T1}$$

Where the only input from the spatial CGE model is the available regional labour supply ($LS_{r,t}$) which can be changed by interregional migration (since demographic changes are not introduce to the modeling framework). All the other parameters variables are previously calibrated or calculated by an external TFP block.

The change of TFP is translated for the SCGE model in a following step:

$$dVA_{r,i,t+1} = \frac{TFP_{r,t+1}}{TFP_{r,t}} \cdot dVA_{r,i,t} \tag{T2}$$

Thus, TFP growth will change the shift parameter of regional industry level value added CES production function (see EQ1 and EQ2). In its simplest form, we assume that regional TFP growth will affect all industries in the region uniformly.

Both of these equations (and the macro block) are determined 'outside' the CGE model between two time periods. Once a static solution is found, TFP change and macro changes will be calculated that will be introduced to the general equilibrium model in the next time period.

2.7.4. Macro model block

The purpose of macro block is to relax the most important macroeconomic closures of the model. Initially government deficit was set to a known initial value (SG_0). Now the macro block will update its annual value according to the rule set by Hungarian laws for the path of government debt.

The accumulation of government debt per GDP is controlled by the following equation:

$$b_t = b_{t-1} + b_{t-1} \cdot (i - \pi - g) + ed_t \tag{M1}$$

Where b_t and b_{t-1} are the government debt (per GDP) in time period t and t-1, i represents nominal interest rate, π is the inflation rate and g is the national growth rate of GDP.

The rule for controlling deficit (and national debt) can be described as follows:

$$ed_t - \overline{ed} = -\beta \cdot \left(b_{t-1} - \overline{b}\right) \tag{M2}$$

Where ed_t is the actual deficit in time period t, $\overline{ed}$ is the target deficit (calibrated from available data), $\overline{b}$ is the target deficit (set to 50%) and β is the sensitivity of adjustment (exogenously fixed in order to provide a smooth path for deficit and debt). The problem with this kind of approach is that the sensitivity of deficit is too strong in the first couple of periods. Thus, we used a slightly modified version of the model. We adjusted the value of β in all time periods accordingly. In the first couple of periods to slow down the adjustment mechanism we set a small positive value which made it possible to reach a smaller deficit than before. Then we start to increase β annually and as we move on with time we set stricter deficit rule.

$$ed_t = \overline{ed} - \beta_{t-1} \cdot \left(b_{t-1} - \overline{b}\right) \tag{M3}$$

With an initial positive value of β government deficit would have been set suddenly from a 3% deficit to a 2% surplus which we found unrealistic thus we slowed down this process by applying an initial negative value for sensitivity. Negative β means that deficit can increase (or at least be higher than the target deficit level) even if debt per GDP is above the target level but because this value is small actual deficit will be less and less than before and with the continuous update of β there will be a smooth convergence to target deficit and debt. The starting value of debt (b_{2010}) is 74.37% and the value of deficit (ed_{2010}) is 3.10%.

Actual equations in the model:

$$ed_{t+1} = \overline{ed} - \beta_t \cdot \left(b_t - \overline{b}\right) \tag{M4}$$

$$b_{t+1} = b_t + b_t \cdot (i - \pi_t - g_t) + ed_{t+1} \tag{M5}$$

Where π_t and g_t is calculated from the SCGE model in each time period. The actual level of the deficit is set by a new equation (which can endogenize variable SG):

$$SG_t = ed_t \cdot adj \cdot \sum_r \sum_i \left(PVA_{r,i} \cdot VA_{r,i}\right)$$

(EQ61 [1])

Where parameter *adj* is responsible to provide consistency between model results and official data. Since the government expenditure and budget calculated from the input-output table does not contain all elements thus deficit will differ from official data. The reason for such differences can be found in the modelling approach. Industries like education have a huge public involvement and yet these industries are still considered to operate as a private industry in the model. Since input-output tables are published in a way that organization (either private or public) are classified into the corresponding industry based on their primary activity. Thus, the consideration of total expenditure and income of the government (within the IO table) does not reflect the official data. Thus, *adj* parameter is calculated to reflect differences between actual deficit and the model based one (*SG0*):

$$adj = {}^{ed_{2010} \cdot GDP_{2010}}/_{SG0} \qquad \text{(M6)}$$

Appendix

A.1. The list of parameters and model variables

The SCGE model

Quantities

$L_{r,j}$	-	Labour demand in region *r* by industry *j*
$K_{r,j}$	-	Capital demand in region *r* by industry *j*
$KI_{r,j}$	-	Capital supply in region *r* to industry *j*
KR_r	-	Capital supply in region *r*
KN	-	Total national capital supply
$VA_{r,j}$	-	Value added in region *r* in industry *j*
$XINT_{r,j}$	-	Total intermediate use by industry *j* in region *r*
$XR_{r,j}$	-	Regional composite output of industry j
$XIM_{r,j}$	-	Industrial (international) import by industry j in region r
$X_{r,j}$	-	Domestic output by industry *j* in region *r*
$XT_{r,j}$	-	Total domestic output by industry *j* in region *r* (with inventories)
$EX_{r,j}$	-	Regional (international) export by industry *j*
$D_{r,j}$	-	Domestic supply of industry *j* in region *r* (without inventories)
$QR_{r,q,j}$	-	Interregional demand for industry j (point of origin: r, destination: q)
$Q_{r,j}$	-	Regional demand for industry *j* in region *r*
$CR_{r,i}$	-	Regional industrial consumption by households
C_r	-	Regional composite household consumption demand
CIM_r	-	Regional import by households
$CTOT_r$	-	Total regional composite consumption
$IR_{r,i}$	-	Regional sectoral investment demand
I_r	-	Regional composite investment demand
IIM_r	-	Regional imported investment
$ITOT_r$	-	Total regional composite investment
$GR_{r,i}$	-	Regional sectoral government demand
G_r	-	Regional composite government demand
GIM_r	-	Regional imported government purchases
$GTOT_r$	-	Total regional composite government demand
$XIR_{r,i,j}$	-	Regional intermediate demand for input *i* by industry *j*
$EXNAT_i$	-	National export by industries

Prices

PL_r	-	Regional price of labour input
$PK_{r,j}$	-	Price of capital input
PKR_r	-	Price of regional capital supply
PKN	-	Price of national capital supply (numeraire)
$PTL_{r,j}$	-	Labour price including taxes

$PTK_{r,j}$	-	Capital price including taxes
$PVA_{r,j}$	-	Price index of value added
$PINT_{r,j}$	-	Price index of total regional intermediate use by industry j
$PXR_{r,j}$	-	Price index of regional output
PIM_r	-	Price index of international import
$PX_{r,j}$	-	Price index of domestic regional output
$PTX_{r,j}$	-	Price index of domestic regional output including production taxes
$PEX_{r,j}$	-	Price index of regional industrial export
$PD_{r,j}$	-	Price index of domestic regional supply
$PQ_{r,j}$	-	Price index of regional final demand by industries
$PCR_{r,j}$	-	Price index of consumption (including consumption-specific commodity tax)
PC_r	-	Price index of domestic total consumption
$PCTOT_r$	-	Price index of total regional consumption
$PIR_{r,j}$	-	Price index of investment (including investment-specific commodity tax)
PI_r	-	Price index of domestic regional investment
$PITOT_r$	-	Price index of total regional investment
$PGR_{r,j}$	-	Price index of government demand (including commodity tax)
PG_r	-	Price index of domestic regional government purchase
$PGTOT_r$	-	Price index of total regional government purchase
$PXIR_{r,j}$	-	Price index of intermediate use (including commodity tax)
$PTEX_{r,j}$	-	Price index of regional export (including commodity tax)
$PEXNAT_j$	-	Price index of national export (in domestic currency)

Nominal variables

YH_r	-	Household income
BH_r	-	Consumption budget
SH_r	-	Household savings
YG	-	Government revenues
$GNAT$	-	Government expenditures
SG	-	Government deficit
$TAXFAC$	-	Tax revenues on factors of production
$TAXPROD$	-	Tax revenues from production taxes
$TAXCOM$	-	Tax revenues from commodity taxes
ER	-	Exchange rate
$INVNAT$	-	Total investment expenditure (total national savings)

Exogenous parameters for the closure of the model

PWM_r	-	World price of import
PWE_j	-	World price of export
$CIV_{r,j}$	-	Domestic changes in inventories in industry *j* in region *r* (with negative sign)
$CIVM_r$	-	Imported changes in inventories in region *r*
LS_r	-	Regional labour supply (updated by interregional migration)
KS_r	-	Regional capital supply (updated by capital accumulation)

SW - Current account

Model parameters

$d_{r,j}^{VA}$ - shift parameter in value added CES (considered as TFP in the SCGE model)
$b_{r,j}^{L}$ - share parameter of labour in value added CES
$b_{r,j}^{K}$ - share parameter of capital in value added CES
$\sigma_{r,j}^{VA}$ - elasticity of substitution in value added CES
$\rho_{r,j}^{VA}$ - elasticity parameter

d_{r}^{KR} - shift parameter in CET
$b_{r,j}^{KR}$ - share parameter of regional industry level capital supply in CET
σ_{r}^{KR} - elasticity of transformation in CET
$\rho_{r,j}^{KR}$ - elasticity parameter

d^{KN} - shift parameter in CET
b_{r}^{KN} - share parameter of regional capital supply in CET
σ^{KN} - elasticity of transformation in CET
ρ^{KN} - elasticity parameter

$a_{r,i,j}$ - input coefficient of input *i* in industry *j*

$v_{r,j}^{VA}$ - input coefficient of value added in Leontief production function
$v_{r,j}^{XINT}$ - input coefficient of intermediate inputs in Leontief production function

$d_{r,j}^{X1}$ - shift parameter in total regional output CES
$b_{r,j}^{XR}$ - share parameter of local output in regional output CES
$b_{r,j}^{XIM}$ - share parameter of industrial import in regional output CES
$\sigma_{r,j}^{X1}$ - elasticity of substitution in total regional output
$\rho_{r,j}^{X1}$ - elasticity parameter

$d_{r,j}^{X2}$ - shift parameter of CET function
$b_{r,j}^{EX}$ - share parameter of export in the CET function
$b_{r,j}^{D}$ - share parameter of domestic supply in the CET function
$\sigma_{r,j}^{X2}$ - elasticity of transportation
$\rho_{r,j}^{X2}$ - elasticity parameter

$d_{r,i}^{Q}$ - shift parameter of the interregional CES demand function
$b_{q,r,i}^{Q}$ - share parameter of interregional demand
$\sigma_{r,i}^{Q}$ - elasticity of substitution in the interregional CES demand function
$\rho_{r,i}^{Q}$ - elasticity parameter

d_{r}^{C} - shift parameter in CES regional domestic consumption demand
$b_{r,i}^{CR}$ - share parameter of sectoral consumption demand in CES
σ_{r}^{C} - elasticity of substitution of sectoral consumption demand
ρ_{r}^{C} - elasticity parameter

d_r^{CTOT}	-	shift parameter in total regional consumption demand CES function
b_r^{C}	-	share parameter of domestic composite consumption
b_r^{CIM}	-	share parameter of import consumption
σ_r^{CTOT}	-	elasticity of substitution in composite CES consumption demand function
ρ_r^{CTOT}	-	elasticity parameter
d_r^{I}	-	shift parameter in total regional investment demand CES function
$b_{r,i}^{IR}$	-	share parameter of sectoral investment demand in CES
σ_r^{I}	-	elasticity of substitution of sectoral investment demand
ρ_r^{I}	-	elasticity parameter
d_r^{ITOT}	-	shift parameter in total regional investment demand CES function
b_r^{I}	-	share parameter of domestic composite investment
b_r^{IIM}	-	share parameter of import investment
σ_r^{ITOT}	-	elasticity of substitution in composite CES investment demand function
ρ_r^{ITOT}	-	elasticity parameter
d_r^{G}	-	shift parameter in total regional government demand CES function
$b_{r,i}^{GR}$	-	share parameter of sectoral government demand in CES
σ_r^{G}	-	elasticity of substitution of sectoral government demand
ρ_r^{G}	-	elasticity parameter
d_r^{GTOT}	-	shift parameter in total regional government demand CES function
b_r^{G}	-	share parameter of domestic composite government demand
b_r^{GIM}	-	share parameter of import government purchases
σ_r^{GTOT}	-	elasticity of substitution in composite CES government demand function
ρ_r^{GTOT}	-	elasticity parameter
d_j^{EXNAT}	-	shift parameter of national export demand CES
$b_{r,j}^{EXR}$	-	share parameter of national sectoral export
σ_j^{EXNAT}	-	elasticity of substitution of national sectoral export
ρ_j^{EXNAT}	-	elasticity parameter
$\tau_{q,r,i}$	-	Iceberg type transportation cost
$tlab_{r,j}$	-	Labour tax rate
$tcap_{r,j}$	-	Capital tax rate
$tprod_{r,j}$	-	Production tax rate
$tcomCR_{r,i}$	-	Commodity tax rate in consumption
$tcomIR_{r,i}$	-	Commodity tax rate investment
$tcomGR_{r,i}$	-	Commodity tax rate government purchases
$tcomXIR_{r,i}$	-	Commodity tax rate in intermediate use
$tcomEX_{r,i}$	-	Commodity tax rate in export
$tcomCIV_{r,i}$	-	Commodity tax subsidies on changes in inventories
sy_r	-	marginal propensity of saving
si_r	-	parameter for distribution of national savings between regions
sg_r	-	parameter for distribution of government expenditures between regions

Dynamic parameters of capital accumulation and migration

δ	-	depreciation rate of capital stock
β	-	parameter for adjustment between the level of investment and capital stock
$LMIGR_{r,t}$	-	net labour migration in region *r* in time period *t*
$NMIGR_{r,t}$	-	net population migration in region *r* in time period *t*
$U_{r,t}$	-	Utility in region *r* in time period *t*
$H_{r,t}$	-	Housing
$N_{r,t}$	-	Population
φ	-	sensitivity of labour migration to utility differences
θ	-	sensitivity of labour migration
$initC_r$	-	initial consumption in the utility function determining labour migration
αH	-	share parameter of housing per capita in the utility function
βH	-	share parameter of consumption per capita in the utility function

The TFP model

$TFP_{r,t}$	-	total factor productivity
$FEDUM_{r,t}$	-	dummy variable for Fejér county
$VADUM_{r,t}$	-	dummy variable for Vas county
$EDUV_{r,t}$	-	Educational investment
$EDUSTOCK_{r,t}$	-	Educational knowledge stock
$PSTCKRT_{r,t}$	-	Patent stock
tfp_const_r	-	constant parameter in the TFP equation
$fedum\_coef_{r,t}$	-	coefficient of dummy variable for Fejér county
$vadum_{coef}{}_{r,t}$	-	coefficient of dummy variable for Vas county
$eduv\_coef_{r,t}$	-	coefficient of educational variables
$emp\_coef_{r,t}$	-	coefficient of employment

The macro model

b_t	-	government debt per GDP
$\overline{b}$	-	long-term desired rate of government debt per GDP
ed_t	-	government deficit per GDP
$\overline{ed}$	-	long-term desired rate of government deficit per GDP
i	-	interest rate (exogenous)
π	-	inflation rate (calculated in the SCGE model)
g	-	GDP growth rate (calculated in the SCGE model)
adj	-	adjustment parameter for consistency between calculated and actual deficit

A.2. List of activities:

- Agriculture (A)
- Mining and quarrying (B)
- Manufacture of food products, beverages and tobacco products (C 10, 11, 12)
- Manufacture of textiles, wearing apparel and leather products (C 13, 14, 15)
- Manufacture of wood and of products of wood, except furniture, paper and paper product and printing and reproduction of recorded media (C 16, 17, 18)
- Manufacture of coke and refined petroleum products (C 19)
- Manufacture of chemicals and chemical products (C 20)
- Manufacture of basic pharmaceutical products and pharmaceutical preparations (C 21)
- Manufacture of rubber and plastic products and other non-metallic mineral products (C 22, 23)
- Manufacture of basic metals and fabricated metal products (C 24, 25)
- Manufacture of computer, electronic and optical products (C 26)
- Manufacture of electrical equipment (C 27)
- Manufacture of machinery and equipment n.e.c. (C 28)
- Manufacture of motorvehicles and other transport equipments (C 29, 30)
- Other manufacturing, repair and installation of machinery and equipment (C 31, 32, 33)
- Electricity, gas, steam and air conditioning supply (D)
- Water collection, treatment and supply (E 36)
- Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services (E 37, 38, 39)
- Construction (F)
- Wholesale and retail trade; repair of motor vehicles and motorcycles (G)
- Transportation and storage (H)
- Accommodation and food service activities (I)
- Publishing activities, motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities (J 58, 59, 60)
- Telecommunications (J 61)
- Computer programming, consultancy and related activities; information service activities (J 62, 63)
- Financial and insurance activities (K)
- Real estate activities (L)
- Legal and accounting activities; activities of head offices; management consultancy activities and architectural and engineering activities; technical testing and analysis (M 69, 70, 71)
- Scientific research and development (M 72)
- Advertising and market research and other professional, scientific activities (M 73, 74, 75)
- Administrative and support service activities (N)
- Public administration and defense; compulsory social security (O)
- Education (P)
- Human health activities (Q 86, 87)
- Social work activities (Q 88)

- Arts, entertainment and recreation (R)
- Other services activities (S)
- Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use (T)
- Activities of extraterritorial organizations and bodies (U)

A.3. Dataset

Local units of the model

In the first step, we need to define the city-regions themselves and the distances between them. We follow the KSH classification in KSH (2014)[1], which introduce the biggest combined settlements in Hungary. According to this, the first table contains the list of regions. There are two types of regions: in the first type, there are city-regions, i.e. the biggest cities and their agglomeration, and in the second one there are county-regions, i.e. the rest of the county. The former is named by the city (with big letters) and the latter is named by the county (with small letters). Most of the cases there are two regions in a county: one is a city-region and the other is the rest of the county. In some cases there are two city-regions in a county but only one region contains the rest of the county. Altogether there are 41 regions in the model. Since our database is built on LAU2 level information we cannot precisely assemble all city-regions but at least we can give a close approximation using our dataset.

BUDAPEST	GYŐR	MISKOLC	PÉCS
EGER	SZOMBATHELY	ZALAEGERSZEG	BÉKÉCSABA
DEBRECEN	DUNAÚJVÁROS	KAPOSVÁR	KECSKEMÉT
NAGYKANIZSA	NYÍREGYHÁZA	SALGÓTARJÁN	SOPRON
SZEGED	SZÉKESFEHÉRVÁR	SZEKSZÁRD	SZOLNOK
TATABÁNYA	VESZPRÉM		
Bács-Kiskun megye	Baranya megye	Békés megye	Borsod-Abaúj-Zemplén megye
Csongrád megye	Fejér megye	Győr-Moson Sopron megye	Hajdú-Bihar megye
Heves megye	Jász-Nagykun-Szolnok megye	Komárom-Esztergom megye	Nógrád megye
Pest megye	Somogy megye	Szabolcs-Szatmár-Bereg megye	Tolna megye
Vas megye	Veszprém megye	Zala megye	

Table 1: List of regions

In order to calculate transportation cost and interregional trade between regions we need to measure the distance between them. We consider three cases, the distance

- inside a region,
- between a city-region and the joint county-region

[1] KSH (2014): Magyarország településhálózata 1. Agglomerációk, településegyüttesek 2014, Budapest

- between all regions except of the regions from the same county.

The following method is used for calculating the distance inside a region. The area of a region is known and it is assumed as a circle, so the averaged radius (R) can be obtained. However, it only shows the average distance between the center and the points of the edge. Instead of this, we use the radius (r), which divide the circle into two equal areas according to figure 1. Thus, radius (r) shows the average distance between any points and the center of the circle, therefore we use (r) as the average distance from the center inside a region. Therefore, it cannot occur that the distance between a region and its neighbor region is smaller than the distance inside a region.

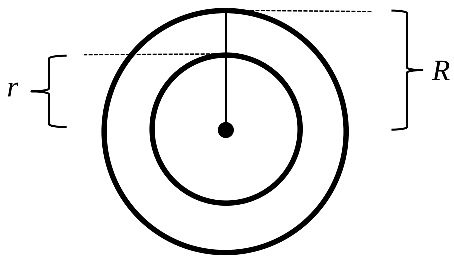


Figure 1: Illustration of distance calculation inside a region

To obtain the distance between a city-region and the joint county-region we take the distance between the centroids of them. In some instance the centroids of these regions are too close to each other (they are almost the same, e.g. Baranya), therefore we use the following method in these cases. We divide the county-region into two areas with equal size and define the centroids of them. Then we take the average distance between the centroid of the city-region and the two new centroids of the county-region weighted by the population.

At last we specify the cross-county distances between the regions. We use the shortest vehicular distance between the centers of the city-regions as the interspace between the city-regions. Similar to this, we use the vehicular distance between the centers of the county-regions (in this case we only use the centroids of the county-regions – one center for one region). The interspace between a city-region and a county-region which are in different counties is also the vehicular distance between the centroids of the regions.

There is an example for the better understanding of the methods of defining distances between the different regions. Figure 2. is an illustration of it.

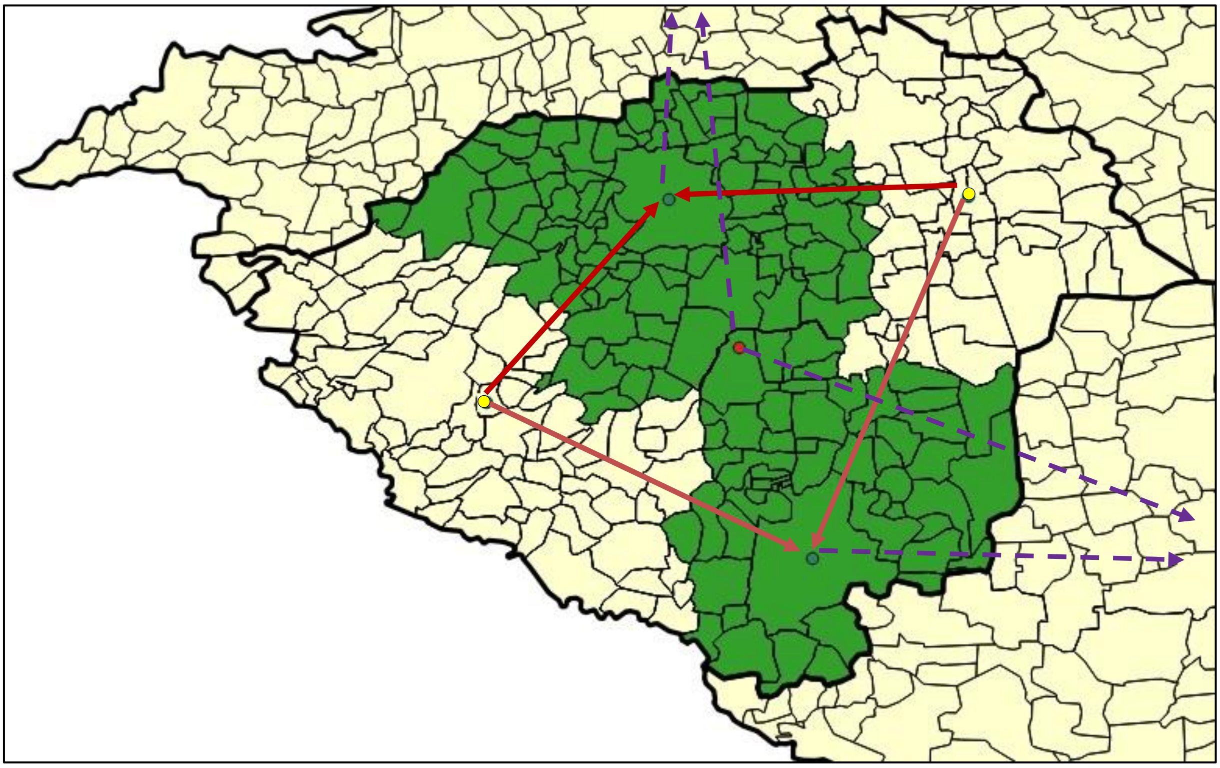
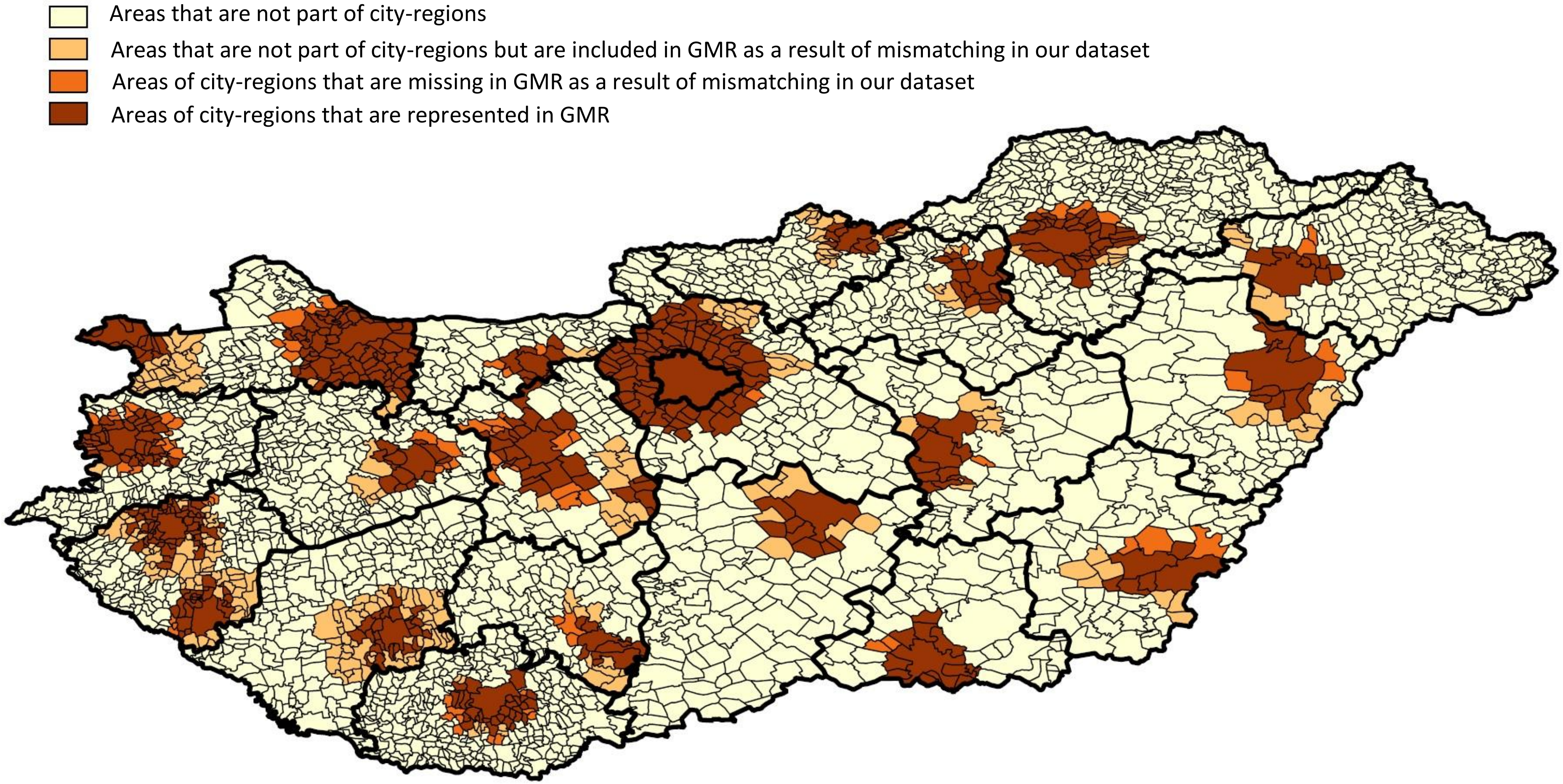


Figure 2: Illustration of the distance definitions.

In the case of Zala County we define two city-regions (two green areas with two green points of their centroids) which divide the rest of the county into two parts. These parts also have a centroid which is shown by a red point in the figure. While determining the distance inside a city-region we only use the area of the region, thus there are no special directions for it on the figure. To specify the distance between the city-regions and the joint county-region which has two separated pieces, we define the two new centroids of the rest of the county (shown by yellow points on the figure). Then we take the average population-weighted distance between the centroid of the city-region and the two centroids of the rest of the county (shown by the same colored arrows). With this method, we get the distance between the city-region and the joint county region. For calculating the interspace between the city-region or the county-region and the other regions outside the county, we take the distance between the centroids of them (shown by purple dashed arrows). In this case, we use the original centroids (shown by red point) of the county regions. In this way, we get the 41x41 size distance matrix.

The actual Hungarian city-regions and their representation in GMR model



A.4. An illustrative example of the interregional SAM

Interregional SAM			Region 1			…	Region 49			ROW (Rest of the world)	Savings	Government (national)	Taxes on production	Taxes on commodities	TLAP	TCAP
			Activities	Production factors	Institutions		Activities	Production factors	Institutions							
			i1 i2 … i39	LAB CAP	HHD GOV INV CIV		i1 i2 … i39	LAB CAP	HHD GOV INV CIV							
Region 1	Activities	i1 i2 … i39	Intermediate goods		Final demand		Intermediate goods		Final demand (interregional import)	Export						
Region 1	Production factors	LAB CAP	Value Added (wage/profit)													
Region 1	Institutions	HHD		Factor income (wage/profit)												
Region 1	Institutions	GOV										Regional gov. expenditure				
Region 1	Institutions	INV									Regional inv. expenditure					
Region 1	Institutions	CIV	Changes in inventories				Changes in inventories			CIV export				Subsidies on CIV		
…																
Region 41	Activities	i1 i2 … i39	Intermediate goods		Final demand (interregional import)		Intermediate goods		Final demand	Export						
Region 41	Production factors	LAB CAP					Value Added (wage/profit)									
Region 41	Institutions	HHD						Factor income (wage/profit)								
Region 41	Institutions	GOV										Regional gov. expenditure				
Region 41	Institutions	INV									Regional inv. expenditure					
Region 41	Institutions	CIV	Changes in inventories				Changes in inventories			CIV export				Subsidies on CIV		
ROW (Rest of the world)			Regional import		Regional import		Regional import		Regional import	Re-export						
Savings					Private savings Gov. savings				Private savings Gov. savings	Foreign savings		Gov. Saving				
Government (national)													TPROD income	TCOM income		
Taxes on production			Taxes on production				Taxes on production									
Taxes on commodities			Taxes on commodities		Taxes on commodities		Taxes on commodities		Taxes on commodities	Taxes on export						
TLAP																

TCAP								

A.5. References

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